PEER- REVIEWED INTERNATIONAL JOURNAL

Aarhat Multidisciplinary International Education Research Journal (AMIERJ)

ISSN 2278-5655

Bi-Monthly

VOL - II | ISSUES - V | [2013]

Chief-Editor:

Ubale Amol Baban

[**Editorial/Head Office**: 108, Gokuldham Society, Dr.Ambedkar chowk, Near TV Towar,Badlapur, MS

PARAMETRIC DIFFERENCE IN TERMS OF CONTIGUOUS RELATIONS.

Dr.Sanjeev Kumar Mishra
M.Sc, B.Ed ,Ph.D (Mathematics)

Abstract:

Here we have defined a parametric difference ($/\Delta$) on a hypergeometric series and found the n^{th} difference of contiguous relations. We have taken special case ${}_2F_1$. This parametric difference also shown in terms of contiguous relations.

1 -Introduction

The idea of extending the number of Parameters in the hypergeometric function seems to have occurred for the first time, in th work of clause(1828). He introduced a series with three numerator parameters and two denominator parameters. . Over the next hundred years the well known set of special summation theorems associated with the names of Soalschutz (1980) dixon (1903) and Dougall's (1907) were developed these are all for series in which A= B+1 and Z=1 . It can be shown that Dougall , s theorem, giving the sum of a ${}_7F_6$ Series, is the most general possible theorem of this kind, the whole theory as it existed then was analysed exhaustively and brought to perfection by W.N.Bailey, in a long series of Papers during the decades of 1920-50. Indeed at this time L.J. Rogers is reported to have said" Nothing remains to be done in the field of hypergeometric series.

The whole theory of the general function ${}_A F_B$ (Z) was still untouched. The first attempts at a general transformation theory were already being made by whipple (1934,1937) and the concept of the asymptotic expansions for the function were already implicit in the work of Barnes (1970a).

2- Formulations:

i. F = F (a,b; c; z)

ii F (1^-) = F (a+1, b+1; c+1; Z)

iii $/\Delta F = F(1^+) - F$

iv. $[(c; a; b); n] = [A, n]$

$$= \frac{(c)_n}{(a)_n \; (b)_n}$$

v. Particular case $[A, o] = 1$

vi. $$F = \sum_{n=0}^{\infty} \frac{[A,n]\, z^n}{n!}$$

vii. $$D \equiv \frac{d}{dz}$$

2.1 Theorem: For hypergeometric function ${}_2F_1\ (a,b; c; z)$

$$/\Delta^n F = \sum_{r=0}^{\infty} [A,r]\,(-1)^{n-r} D^r F$$

Proof: Let

$$/\Delta F = F(a+1\ b+1;c;z) - F(a,b; c; z)$$

$$/\Delta \equiv [A, 1]\, D - [A,0]$$

$$/\Delta^2 F = /\Delta\,[\Delta F]$$

$$/\Delta^2 \equiv [A, 1]\, D^2 - {}^2c_1[A, 1]\, D + {}^2c_2\, [A, 0]\, D^0$$

$$/\Delta^3 \equiv [A, 3]\, D^3 - {}^3c_1[A, 2]\, D^2 + {}^3c_2\, [A, 1]\, D - [A,0]$$

Similarly

$$/\Delta^n \equiv [A, n]\, D^n - {}^nc_1[A, n-1]\, D^{n-1} + {}^nc_2\, [A, n-2]\, D^{n-2} - \ldots\ldots + (-1)^n[A,0]$$

Or

$$/\Delta^n \equiv \sum_{r=0}^{n} [A,r]\,(-1)^{n-r} D^r$$

Hence

$$/\Delta^n F = \sum_{r=0}^{\infty} [A,r]\,(-1)^{n-r} D^r F$$

Contigous relations:

3.1 Theorem: To prove that $/\Delta^n F(a, b; c; z) = F[n^+] - {}^nc_1 F[(n-1)^+] + {}^nc_2 F[(n-2)^+] - {}^nc_3 F[(n-3)^+] + \ldots\ldots + (-1)^n F$

Proof:

$$/\Delta F(a, b; c; z) = F[1^+] - F$$

$$/\Delta^2 F = F[2^+] - 2F[1^+] + F$$

$$/\Delta^3 F = F[3^+] - 2F[2^+] + 3F[1^+] - F$$

$$/\Delta^4 F = F[4^+] - 4F[3^+] + 6F[2^+] - 4F[1^+] + F$$

Similarly

$$/\Delta^n F(a, b; c; z) = F[n^+] - {}^nc_1 F[(n-1)^+] + {}^nc_2 F[(n-2)^+] - {}^nc_3 F[(n-3)^+] + \ldots\ldots + (-1)^n F$$

Reference

Andrews G.E. Baxler R.J.& Forrester P.J. [1984] "Eight vertex SOS Model and generalized Rogers –Ramanujan type identities" , Jour Statistical Phys. 35, 193-266.

Andrews G.E.[1966](i) Ramanujan – Gordon indentities " , Amer., J. Math., 88,884-846 (ii) An analytic generalization of a Roger's Ramanujan identities for odd modulii" , Proc, Nat.Acd. Sci, 7, 4082-85.

Bailey W.N.[1929](i) "Some identities involving generalized hypergeometric series" Proc. London Math, Soc .29,503-516.

ii) "Some indentities in combinatory anlaysis"Proc. London. Math.Soc.(2) 49,421-435.

Bateman Harry "Higher transcendental functions Vol. I,II And III. Mcgraw – Hill Book company .Inc.

Exton Harold(i) Triple Gaussia Hypergeometric functions- I Indian J.Pune,Appl. Math., 25(10) , 1073-1079 ii)"Triple Gaussian hypergeometric functions II" Indian J.Pure.Appl.Math., 26(8) ,807-8011

Slater L.J.[1951](i) "A new proof of Roger' transformations Of infinite series. Proc. London. Maths. Soc.(2), 54,147-67.(ii) "Further identities of the Rogers Ramanujan type" Proc. London .Math. Soc.(2),54,147-67.

Singh Anand & Dhami H.S.[1999]“construction of generalized multiple Hypergeometric functions and contrivance of Appell functions as its special cases”. I.M.A. preprint series# 1619, University of Minnesota, Minneapolis , U.S.A.