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INEQUALITIES ON NUMBER THEORETIC FUNCTIONS

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Abstract

We have observed the inequalities on number theoretic functions and one can easily find out the maximum and minimum fluctuations in a defined range.

1. Introduction:

The theory of numbers in that branch of Mathematics which deals with properties of whole number, 1,2,3,4,5,6,---------- also called the counting numbers, or positive integers. The positive integers are undoubtedly man's first mathematical creation. It is hardly possible to imagine human beings without the ability to count, at least within a limited range. Historical record shows that's early as 5700 B.C., the ancient Sumerians kept a Calendar, so they must have developed some from arithmetic.

When ancient civilizations reached a level which provided leisure time to ponder about things, some people begin to speculate about the nature and properties of numbers. This curiosity developed in to a sort of number mysticism or numerology, and even today numbers such as 3,7,11, and 13 are considered omens of good or bad luck.

The professional mathematician is attracted to number theory because the way all the weapons of modern mathematics can brought to bear on its problems. As a matter of fact, many important branches of mathematics had their origin in number theory. New problem arise more rapidly and many of the old problems have remained unsolved for centuries.

2. Formulations:

$\Delta d(x) = d(x) - d(x+1), \quad x \geq 1$ ……..2.1

Where d(x) denotes divisor function:

$\Delta \mu(x) = \mu(x) - \mu(x+1), \quad x \geq 1$ ……..2.2

Where $\mu(x)$ denotes mobius function:

$\Delta \varphi(x) = \varphi(x) - \varphi(x+1), \quad x \geq 1$ ……..2.3

Where $\varphi(x)$ denotes Eulers totient function:

$\Delta\sigma(x) = \sigma(x) - \sigma(x+1), \quad x \geq 1$ ……..2.4

Where $\sigma(x)$ denotes sum of the divisors of x:

3. Experiment:

For the numbers 1 to 100 a table has been formulated for (2.1) and following results have been obtained:

4. Theorem: With the help of table 4 (c)

If $\Delta d = k$ then

$$\prod_{i=1}^{n}(\alpha_i + 1) = \prod_{j=1}^{m}(\beta_i + 1) + K$$

Where $x = \prod_{i=1}^{n} p_i^{\alpha_i}$ and $x + 1 = \prod_{j=1}^{m} q_j^{\beta_j}$

In particular

If K = 0

then $\Delta d = 0$

$\Rightarrow$ x, x+1 have similar factorization

Proof can be easily seen with the help of table 4(c)

5. Experiment: For the numbers 1 to 100 a table has been formulated for (2.2) and following results have been obtained:

Theorem 5.1:

Value	Behaviour for x, x+1	
	$x = \prod_{i=1}^{n} p_i^{\alpha_i}$	$x + 1 = \prod_{j=1}^{m} q_j^{\beta_j}$
-2	$\alpha_i = 1 \forall i$ K = odd	$\beta_j = 1 \forall j$ I = even
-1	$\alpha_i = 1 \forall i$ K = odd	For at least one value of j, $\beta_j > 1$
	For at least one value of i, $\alpha_i > 1$	$\beta_j = 1 \forall j$ I = even

$\mu(x)=$

0	For at least one value of i, $\alpha_i>1$	For at least one value of j, $\beta_j>1$
	$\alpha_i=1\forall$ i K = odd	$\beta_j=1\forall$j I = odd
1	$\beta_j=1\forall$j I = even	For at least one value of j, $\beta_j>1$
	For at least one value of j, $\beta_j>1$	$\alpha_i=1\forall$ i K = odd
2	$\beta_j=1\forall$j I = even	$\alpha_i=1\forall$ i K = odd

Proof can be easily seen with the help of table 4(d)

6. Experiment: For the number 1 to 100 a table has been formulated for 2.3 and following results have been obtained.

Group Size	Maximum fluctuation
1	64
2	118
4	118
5	18
10	122
20	126
25	132
50	132
100	132

Table 4(a)

Theorem 7.0 If φ denotes Eulers function then $\Delta\varphi$ is increasing the interval.

Proof can be easily seen with the help of table 4(a) and 4 (f)

7.1 Experiment: With the help of table 4(h) for x (1 to 200) by taking the group size as divisor of x (e.g. 200) 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200 following observations have been made:

Group Size	Maximum fluctuation
1	364
2	730
4	730
5	730
8	730
10	730
20	730
25	730
40	730
50	730
100	730
200	730

Table 4(b)

Theorem 8.0 If σ denotes sum of the divisors then $\Delta\sigma$ have maximum fluctuation at group size one and remains constant in other intervals.

Proof can be easily seen with the help of tables 4(b), 4(g) and 4(h).

$$d(x) = \frac{\sum 1}{d/x}$$

X	d(x)	Δ d(x)	**x**	d(x)	Δ d(x)	**x**	d(x)	Δ d(x)
1	1	-1	**36**	9	7	**71**	2	-9
2	2	0	**37**	2	-2	**72**	11	9
3	2	-1	**38**	4	2	**73**	2	-2
4	3	1	**39**	2	-6	**74**	4	-2
5	2	-2	**40**	8	6	**75**	6	0
6	4	2	**41**	2	-6	**76**	6	2
7	2	-2	**42**	8	6	**77**	4	-3
8	4	1	**43**	2	-4	**78**	7	5
9	3	-1	**44**	6	0	**79**	2	-7
10	4	2	**45**	6	2	**80**	9	4
11	2	-4	**46**	4	2	**81**	5	1
12	6	4	**47**	2	-7	**82**	4	2

13	2	-2	**48**	9	7	**83**	2	-8
14	4	0	**49**	2	-3	**84**	10	6
15	4	-1	**50**	5	3	**85**	4	0
16	5	3	**51**	2	-3	**86**	4	0
17	2	-4	**52**	5	3	**87**	4	-4
18	6	4	**53**	2	-5	**88**	8	6
19	2	-4	**54**	7	3	**89**	2	-10
20	6	2	**55**	4	-4	**90**	12	10
21	4	0	**56**	8	4	**91**	2	-4
22	4	2	**57**	4	0	**92**	6	2
23	2	6	**58**	4	2	**93**	4	0
24	8	5	**59**	2	-10	**94**	4	0
25	3	-1	**60**	12	10	**95**	4	-8
26	4	0	**61**	2	-2	**96**	12	10
27	4	-2	**62**	4	-1	**97**	2	-4
28	6	4	**63**	5	-2	**98**	6	0
29	2	-6	**64**	7	-2	**99**	6	0
30	8	6	**65**	4	-2	**100**	9	7
31	2	-3	**66**	6	4			
32	5	3	**67**	2	-4			
33	2	-2	**68**	6	2			
34	4	0	**69**	4	-4			
35	4	-5	**70**	8	6			

Table 4(c)

x	μ(x)	Δ μ(x)	**x**	μ(x)	Δ μ(x)	**x**	μ(x)	Δ μ(x)
1	1	2	**34**	1	0	**67**	-1	
2	-1	0	**35**	1	1	**68**	0	1
3	-1	-1	**36**	0	1	**69**	-1	0
4	0	1	**37**	-1	-2	**70**	-1	
5	-1	-2	**38**	1	0	**71**	-1	-1
6	1	2	**39**	1	1	**72**	0	1
7	-1	-1	**40**	0	1	**73**	-1	-2
8	0	0	**41**	-1	0	**74**	1	1
9	0	-1	**42**	-1	0	**75**	0	0
10	1	2	**43**	-1	-1	**76**	0	-1
11	-1	-1	**44**	0	0	**77**	1	2
12	0	1	**45**	0	-1	**78**	-1	0
13	-1	-2	**46**	1	2	**79**	-1	-1

14	1	0	**47**	-1	-1	**80**	0	0
15	1	1	**48**	0	0	**81**	0	-1
16	0	1	**49**	0	0	**82**	1	2
17	-1	-1	**50**	0	-1	**83**	-1	-1
18	0	1	**51**	1	1	**84**	0	-1
19	1	-1	**52**	0	1	**85**	1	0
20	0	-1	**53**	-1	-1	**86**	1	0
21	1	0	**54**	0	-1	**87**	1	1
22	1	2	**55**	1	1	**88**	0	1
23	-1	-1	**56**	0	1	**89**	-1	-1
24	0	0	**57**	-1	-2	**90**	0	1
25	0	-1	**58**	1	2	**91**	-1	-1
26	1	1	**59**	-1	-1	**92**	0	-1
27	0	0	**60**	0	1	**93**	1	0
28	0	1	**61**	-1	-2	**94**	1	2
29	-1	0	**62**	1	2	**95**	-1	-1
30	-1	0	**63**	-1	-1	**96**	0	1
31	-1	-1	**64**	0	1	**97**	-1	-1
32	0	-1	**65**	-1	0	**98**	0	0
33	1	0	**66**	-1	0	**99**	0	0
						100	0	1

Table 4 (d)

X	φ(x)	Δ φ(x)	**x**	φ(x)	Δ φ(x)	**x**	φ(x)	Δ φ(x)
1	1	0	**35**	24	12	**69**	44	20
2	1	-1	**36**	12	-24	**70**	24	-46
3	2	0	**37**	36	18	**71**	70	46
4	2	-2	**38**	18	-6	**72**	24	-36
5	4	2	**39**	24	-12	**73**	72	-48
6	2	-4	**40**	16	-24	**74**	36	-4
7	6	2	**41**	40	28	**75**	40	4
8	4	-2	**42**	12	-30	**76**	36	-4
9	6	2	**43**	42	22	**77**	60	36
10	4	-6	**44**	20	-4	**78**	24	-54
11	10	6	**45**	24	2	**79**	78	46
12	4	-8	**46**	22	-24	**80**	32	-22
13	12	6	**47**	46	30	**81**	54	14
14	6	-2	**48**	16	-26	**82**	40	-42

15	8	0	**49**	42	22	**83**	82	58
16	8	-8	**50**	20	-12	**84**	24	-40
17	16	10	**51**	32	8	**85**	64	22
18	6	-12	**52**	24	-28	**86**	42	-14
19	18	10	**53**	52	34	**87**	56	16
20	8	-4	**54**	18	-22	**88**	40	-48
21	12	2	**55**	40	16	**89**	88	64
22	10	-12	**56**	24	-12	**90**	24	-48
23	22	14	**57**	36	8	**91**	72	28
24	8	-12	**58**	28	-30	**92**	44	-16
25	20	12	**59**	58	42	**93**	60	14
26	12	-6	**60**	16	-44	**94**	46	-68
27	18	6	**61**	60	30	**95**	72	40
28	12	-14	**62**	30	-6	**96**	32	-64
29	28	20	**63**	36	4	**97**	96	54
30	8	-22	**64**	32	-16	**98**	42	-18
31	30	14	**65**	48	28	**99**	60	20
32	16	-4	**66**	20	46	**100**	40	-60
33	20	4	**67**	66	34			
34	16	-8	**68**	32	-12			

Table 4(e)

$\Delta\varphi$ (x)

G.S.	F	G.S.	F	G.S.	F	G.S.	F	G.S.	F
0-2	1	**58-60**	86	**28-32**	42	**50-55**	62	**0-25**	26
2-4	2	**60-62**	74	**32-36**	36	**55-60**	86	**25-50**	60
4-6	6	**62-64**	20	**36-40**	42	**60-65**	74	**50-75**	94
6-8	6	**64-66**	62	**40-44**	58	**65-70**	92	**75-100**	132
8-10	8	**66-68**	58	**44-48**	56	**70-75**	94	**0-50**	60
10-12	14	**68-70**	66	**48-52**	50	**75-80**	100	**50-100**	132
12-14	14	**70-72**	92	**52-56**	62	**80-85**	98		
14-16	8	**72-74**	44	**56-60**	86	**85-90**	112		
16-18	22	**74-76**	8	**60-64**	74	**90-95**	108		
18-20	22	**76-78**	90	**64-68**	62	**95-100**	118		
20-22	14	**78-80**	100	**68-72**	92	**0-10**	8		
22-24	26	**80-82**	56	**72-76**	52	**10-20**	22		
24-26	24	**82-84**	100	**76-80**	100	**20-30**	42		
26-28	20	**84-86**	62	**80-84**	100	**30-40**	42		

28-30	42	**86-88**	64	**84-88**	70	**40-50**	60
30-32	36	**88-90**	112	**88-92**	112	**50-60**	86
32-34	12	**90-92**	76	**92-96**	108	**60-70**	92
34-36	36	**92-94**	82	**96-100**	118	**70-80**	100
36-38	42	**94-96**	108	**0-5**	4	**80-90**	112
38-40	18	**96-98**	118	**5-10**	8	**90-100**	122
40-42	58	**98-100**	80	**10-15**	14	**0-20**	22
42-44	52	**0-4**	2	**15-20**	22	**20-40**	44
44-46	26	**4-8**	6	**20-25**	26	**40-60**	86
46-48	56	**8-12**	14	**25-30**	42	**60-80**	100
48-50	48	**12-16**	14	**30-35**	36	**80-100**	126
50-52	36	**16-20**	22	**35-40**	42		
52-54	62	**20-24**	26	**40-45**	58		
54-56	38	**24-28**	26	**45-50**	56		
56-58	38						

Table4 (f)

	σ(x)	**Δσ(x)**	**x**	**σ(x)**	**Δσ(x)**	**x**	**σ(x)**	**Δσ(x)**
1	1	-2	**35**	48	-43	**69**	96	-48
2	3	-1	**36**	91	53	**70**	144	72
3	4	-3	**37**	38	-22	**71**	72	-123
4	7	1	**38**	60	4	**72**	195	121
5	6	-6	**39**	56	-34	**73**	74	-40
6	12	4	**40**	90	48	**74**	114	-10
7	8	-7	**41**	42	-54	**75**	124	-16
8	15	2	**42**	96	52	**76**	140	44
9	13	-5	**43**	44	-40	**77**	96	-72
10	18	6	**44**	84	6	**78**	168	88
11	12	-16	**45**	78	6	**79**	80	-106
12	28	14	**46**	72	24	**80**	186	65
13	14	-10	**47**	48	-76	**81**	121	-5
14	24	0	**48**	124	67	**82**	126	42
15	24	-7	**49**	57	-36	**83**	84	-140
16	31	13	**50**	93	21	**84**	224	116
17	18	-21	**51**	72	-26	**85**	108	-24
18	39	19	**52**	98	44	**86**	132	12
19	20	-22	**53**	54	-66	**87**	120	-60
20	42	10	**54**	120	48	**88**	180	90
21	32	-4	**55**	72	-48	**89**	90	-144
22	36	12	**56**	120	40	**90**	234	122

23	24	-36	**57**	80	-10	**91**	112	-56
24	60	29	**58**	90	30	**92**	168	40
25	31	-11	**59**	60	-108	**93**	128	-16
26	42	2	**60**	168	106	**94**	144	24
27	40	-16	**61**	62	-34	**95**	120	-132
28	56	26	**62**	96	-8	**96**	252	154
29	30	-42	**63**	104	-23	**97**	98	-23
30	72	40	**64**	127	43	**98**	121	-35
31	32	-31	**65**	84	-60	**99**	156	-61
32	63	15	**66**	144	76	**100**	217	115
33	48	-6	**67**	68	-58			
34	54	6	**68**	126	30			

x	**σ(x)**	**Δσ(x)**	**x**	**σ(x)**	**Δσ(x)**	**x**	**σ(x)**	**Δσ(x)**
100	217	15	**137**	288	148	**174**	372	102
101	102	-114	**138**	140	Δ196	**175**	270	90
102	216	112	**139**	336	144	**176**	180	-366
103	104	-106	**140**	192	-24	**177**	546	364
104	210	18	**141**	216	48	**178**	182	-154
105	192	30	**142**	168	-235	**179**	336	88
106	162	54	**143**	403	223	**180**	248	-112
107	108	-172	**144**	180	-42	**181**	360	132
108	280	170	**145**	222	-6	**182**	228	156
109	110	-106	**146**	228	-38	**183**	384	196
110	216	64	**147**	266	-116	**184**	188	-148
111	152	-96	**148**	150	-222	**185**	336	16
112	248	134	**149**	372	220	**186**	320	-40
113	114	-126	**150**	152	-148	**187**	360	168
114	240	96	**151**	300	66	**188**	192	-316
115	144	-66	**152**	234	-54	**189**	508	314
116	210	28	**153**	288	96	**190**	194	-100
117	182	2	**154**	192	-200	**191**	294	-42
118	180	36	**155**	392	234	**192**	336	-63
119	144	-216	**156**	158	-82	**193**	399	201
120	360	227	**157**	240	24	**194**	198	-270
121	133	-53	**158**	216	-162	**195**	468	268
122	186	18	**159**	378	186	**196**	200	-265
123	168	-56	**160**	192	-171	**197**	465	193
124	224	68	**161**	363	199			
125	312	184	**162**	294	6			

126	312	184	**163**	294	6
127	255	79	**164**	252	84
128	176	-160	**165**	168	-312
129	336	204	**166**	480	297
130	132	-204	**167**	183	-177
131	336	176	**168**	360	100
132	160	-44	**169**	260	-48
133	204	-36	**170**	308	134
134	240	-30	**171**	174	-186
135	270	132	**172**	360	112
136	138	-150	**173**	248	-124

Table 4(g)

Δσ(x)

G.S	F	G.S.	F	G.S.	F	G.S.	F	G.S.	F	G.S.	F
0-2	1	**68-70**	120	**134-136**	168	**0-4**	4	**132-136**	220	**75-80**	194
2-4	4	**70-72**	244	**136-138**	298	**4-8**	11	**136-140**	344	**80-85**	256
4-6	10	**72-74**	161	**138-140**	344	**8-12**	30	**140-144**	458	**85-90**	266
6-8	11	**74-76**	60	**140-142**	168	**12-16**	24	**144-148**	265	**90-95**	254
8-10	11	**76-78**	160	**142-144**	458	**16-20**	41	**148-152**	442	**95-100**	286
10-12	30	**78-80**	194	**144-146**	265	**24-28**	45	**152-156**	434	**100-105**	226
12-14	24	**80-82**	70	**146-148**	154	**28-32**	82	**156-160**	370	**110-115**	260
14-16	20	**82-84**	256	**148-150**	442	**36-40**	87	**164-168**	609	**115-120**	443
18-20	41	**86-88**	150	**150-152**	368	**40-44**	106	**172-172**	474	**120-125**	383
20-22	16	**88-90**	266	**152-154**	150	**44-48**	143	**176-180**	730	**130-135**	408
22-24	65	**90-92**	178	**154-156**	434	**52-56**	114	**180-184**	518	**135-140**	344
26-28	42	**94-96**	286	**156-158**	316	**56-60**	214	**188-192**	630	**145-150**	442
28-30	82	**96-98**	189	**158-160**	348	**60-64**	140	**192-196**	414	**150-155**	420
30-32	71	**98-100**	176	**160-162**	370	**64-68**	136	**196-200**	414	**150-155**	420
32-34	21	**100-102**	226	**162-164**	329	**68-72**	244	**0-5**	7	**160-165**	370
34-36	75	**102-104**	218	**164-166**	78	**72-76**	161	**5-10**	11	**165-170**	609
36-38	75	**106-108**	342	**166-168**	609	**80-84**	256	**15-20**	30	**170-175**	320
40-42	106	**108-110**	276	**168-170**	474	**84-88**	176	**20-25**	65	**175-180**	730
42-44	92	**110-112**	230	**170-172**	182	**88-92**	266	**25-30**	82	**185-189**	344
44-46	18	**112-114**	260	**172-174**	320	**92-96**	286	**30-35**	83	**190-195**	630
46-48	143	**114-116**	162	**174-176**	236	**96-100**	215	**35-40**	96	**195-200**	538
48-50	103	**118-120**	443	**176-178**	468	**104-108**	342	**40-45**	106	**0-8**	11
50-52	114	**120-122**	280	**178-180**	730	**108-112**	276	**45-50**	143	**8-16**	29
54-56	96	**122-124**	124	**180-182**	518	**112-116**	260	**50-55**	114	**16-24**	65

56-58	50	**124-126**	340	**182-184**	244	**116-120**	443	**55-60**	214	**24-32**	82
58-60	214	**126-128**	311	**184-186**	264	**120-124**	283	**60-65**	166	**32-40**	96
60-62	140	**128-130**	364	**186-188**	344	**124-128**	340	**65-70**	136	**40-48**	143
62-64	66	**130-132**	408	**188-190**	208	**128-132**	408	**70-75**	244	**48-56**	133
64-66	136	**132-134**	220	**190-192**	630						
66-68	134	**150-160**	434	**192-194**	414						
66-64	214	**160-170**	609	**194-196**	264						
64-72	244	**170-180**	730	**196-198**	538						
72-80	227			**198-200**	233						

80-88	256	**180-190**	518	**0-100**	298
88-96	298	**190-200**	630	**100-200**	730
96-104	268				
104-112	342	**0-20**	41		
112-120	443	**20-40**	96		
120-128	383	**40-60**	214		
128-136	408	**60-80**	244		
136-144	458	**80-100**	298		
144-152	445	**100-120**	443		
152-160	434	**120-140**	431		
160-168	609	**140-160**	469		
168-176	483	**160-180**	730		
176-184	730	**180-200**	680		
184--192	630	**0-25**	65		
192-200	584	**25-50**	143		
0-10	13	**50-75**	244		
10-20	41	**75-100**	298		
20-30	82	**100-125**	383		
30-40	96	**125-150**	462		
40-50	143	**150-175**	609		
50-60	214	**175-200**	730		
60-70	166	**0-40**	96		
70-80	244	**40-80**	244		
80-90	266	**80-120**	443		
90-100	286	**120-160**	469		
100-110	342	**160-200**	730		
110-120	443	**0-50**	143		
120-130	383	**50-100**	298		
130-140	408	**100-150**	462		
140-150	458	**150-200**	730		

Table 4 (h)

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