

**ANALYSIS AND SIMULATION OF A SEIR MODEL FOR COVID-19 VIRUS IN PUNE****Sagar Lahanu Khairnar,***Department of Mathematics, Changu Kana Thakur Arts, Commerce and Science College, New Panvel (Autonomous)***Abstract:**

In this paper we have considered the mathematical time delay SEIR (Susceptible-Exposed-Infectious-Recovered) model. We simulated our results by using MATLAB software and forecasted the number of infected cases and compared the actual data and forecasted results.

Keywords: SEIR, Mathematical modelling, time delay SEIR

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Introduction:

The famous SIR mathematical model first developed by Kermack-McKendrick (1927). Since then, many researchers modified and improved the SIR model. In this article we used SEIR model with time delay and by using MATLAB we simulated the infection situation in PUNE city.

We have considered the following SEIR model

$$\begin{aligned}\dot{S}(t) &= -\lambda \frac{S(t)I(t)}{N(t)} + \gamma I(t - \tau) \\ \dot{E}(t) &= \lambda \frac{S(t)I(t)}{N(t)} - \lambda \frac{S(t - \omega)I(t - \omega)}{N(t - \omega)} \\ \dot{I}(t) &= \lambda \frac{S(t - \omega)I(t - \omega)}{N(t - \omega)} - (\gamma + \epsilon)I(t) \\ \dot{R}(t) &= \gamma I(t) - \epsilon I(t - \tau)\end{aligned}$$

The above simultaneous nonlinear normal differential equation system represents the SEIR model.

Where, t: hours (days)

Top dots: fractions of time t

$S(t)$: Sensitive population at time t (number of uninfected people)

$E(t)$: Population exposed to the virus but not infected at time t

$I(t)$: Infected population at time t

$R(t)$: Recovery population at time t (population recovered from disease after infection)

$N(t)$: Total population at time t, $N(t) = S(t) + E(t) + I(t) + R(t)$

$\lambda \geq 0$: Infection, $\gamma \geq 0$: Recovery rate, $\epsilon \geq 0$: Viral mortality, τ : Time to gain immunity (days), ω : Delay time (days) before developing the disease

In this model, the total population will continue to decrease unless the infected population becomes zero. In other terms, the death toll $D(t)$ is $N(0)$ as the initial population at $t=0$.

$$\dot{N}(t) = -\epsilon I(t)$$

$$D(t) = N(0) - N(t) = N(0) - (S(t) + E(t) + I(t) + R(t))$$

Prerequisite for calculation:

We have not considered the natural mortality rate natural fertility rate because it is a relatively short simulation (about one year).

The same virus does not contract twice (after τ days it acquires immunity and does not suffer again).

In this model we have taken the important number of the mathematical engineering of infectious diseases

as $R_0 = \frac{\lambda}{\gamma + \epsilon}$

If $R_0 > 1$ then situation becomes it becomes a pandemic otherwise it is endemic

Literature Review

Deguen and et.al. (2000) Syafruddin (2012) provided the prediction of the Dengue cases in Selangor Malaysia by using SEIR model. Zhang and et.al. (2006) Rachah (2018) studied the EBOLA pandemic in Liberia by using SEIR model. Yan and et.al. (2006) studied the special case of the SIER model with zero latent period, and concluded that $R_0 > 1$ leads to the global stability of the endemic equilibrium, which provided the answers to the conjecture proposed by Diekmann and Heesterbeek. Lekone and Finkenstädt (2006) studied an outbreak of Ebola in the Democratic Republic of Congo in 1995 by using SIER model. Gao (2008) determined that the large vaccination rate or short pulse of vaccination or long latent period is adequate condition for the extinction of the disease. Dukic and et.al. (2012) studied the Google Flu Trends in United States of America during 2003-2009. Dubey and et.al. (2013) proposed SEIR epidemic model for treatment of infectives considering the development of acquired immunity in recovered individuals and concluded that the existence of unique endemic equilibrium depends on the basic reproductive number R_0 and on treatment rate. Saito and et.al. (2013) analysed varicella epidemics in France by using SEIR model. Shah and Gupta (2013) used SIER models to analysed the **vector borne diseases**. Gao and et.al. (2013) shown that the accuracy of the prediction of number of cases in Bangladesh in the integrated model is more than the general SEIR model. Mojeeb and et.al. (2017) provided the simple and continuous-time linear vaccination-based control strategy for a SEIR disease propagation model.

A modified, deterministic SEIR model used by Diaz and et.al. (2018) to study 2014 pandemic of EBOLA in south Africa. Getz and et.al. (2019) considered the time dependent SIER model to study Ebola pandemic in West Africa of 2014–2015. Teles (2020) converted the four dimensional the SIER model into three dimensional SIER model with limit condition by using the change of variable. Lacitignola and et.al. (2021) studied Z-controlled approach and studied the pandemic of Italy and New Zealand. Piovella and et.al. (2020) studied the covid 19 pandemic by using seir model in python. Peiliang and et.al. (2020), Kounchev (2020), presented a spline-based SEIR model with Time-varying Beta and Gamma parameters, (TVBG-SEIR) model, estimated the practical implications of the public health interventions and measures. Zha and et.al. (2020) explored the optimal prevention and control measures for varicella pandemic in a school of

China by using dynamic model. Youssef and et.al. (2020) used SEIR model to predict the number of cases in South East Asia. Hui and et.al. (2020) used SEIR model for the short-term prediction of the covid 19 cases in China. Yang and et.al. (2020) predicted pandemic by using SEIR model and Artificial Intelligence in China. Tang and et.al. (2020) predicted and analyzed the trend of the epidemic situation and estimated the parameters involved in the infection dynamics model (SIER), and then simulated the established dynamic equations based on public data by using MATLAB. Rădulescu and et.al. (2020) made predictions, and assess the efficiency of control measures like lockdown and social distancing etc. , in a sustainability context in New York City. Intissar and et.al. (2020) concluded the disease-free equilibrium point is locally as well as globally asymptotically stable by using SEIR model. Putra and et.al. (2020), Annas (2020) studied Stability Analysis and numerical simulation of the SEIR model and predicted the number of COVID-19 in Indonesia.

Bhupkar and et.al. (2020) used graph theory approach and forecasted the number of covid-19 infected persons in the all sates. They considered the growth type of graph with three variations namely 1-1,1-p and 1-all. Yadav and et.al. (2020) studied the current status of the pandemic with the global data by using SIER model and R-software. Bajaj and et.al. (2020) used the supervise machine learning algorithm and time series analysis of the Mira-Bhayander and Akola Municipal corporations. Saxena studied the pandemic conditions by using random theory / mathematical models and data science / machine learning techniques. Sahoo and et.al. (2020) used a data driven epidemic model to forecast the cases of infected, recovered and death due to covid-19 in India. Prasad (2020) used data first approach and time dependent SIR model to forecast the total number of infected people in different countries. Shaikh and et.al.(2020) estimated the future outbreak of covid-19 and potential control strategies using Fractional differential mathematical model in some states of India. Darapaneni and et.al. (2020) predicted and analysed the number of covid-19 cases in Maharashtra and Delhi by using ARIMA and SEIRD model. Sahasranaman and et.al. (2021) studied the spread of COVID-19 across metropolitan cities in Asia and Africa. Wagh and et.al. (2020) used SEIR model to forecast the he number of deaths due to covid-19 in india. Rajesh and et.al. (2020) used SIR(D) model to predict the future of the epidemic in India and also studied the effect of lock-down/social isolation on the model. Pujari and et.al. (2020) used SIR model to study the epidemic situation of India. Mahalle and et.al. (2020) presented different predictive analytics techniques available for trend analysis, different models and algorithms, and their comparison and predicted the number of covid-19 cases by using Prophet algorithm. Welling and et.al. (2021) used MIMANSA (Multilevel Integrated Model with a Novel Systems Approach) to study the effect of vaccination on the number of COVID cases in the USA. Dixit (2021) proposed a new Advanced SIER model which has greater accuracy of the reproduction number. Atkeson (2021) forecasted the evolution of the epidemic for the next two years with continuing seasonality, pandemic fatigue, and spread of the new variant in United States and the United Kingdom by using SIER model. López and et.al. (2021) used modified *SEIR* compartmental model for the prediction of covid 19 cases. Efimov (2021)analysed the pandemic situations in 8 countries by using SIER model. Rapolu and et.al. (2022) used a data-driven model SEIRD predicted infected, recovered, and deceased rates of COVID-19 for the 21 days.

Objective

1. To study the COVID-19 pandemic in Pune City.
2. To calculate the number of reproductions.

Research Methodology :

We have considered the initial time is as of 30th June, 2020 (t=0), 8th October, 2020 (t = 100, Day 100 from the initial time), 16th January, 2021(t=200), and 26th April, 2021 (t=300).

Basic data on the infected population of 10305 people, 752 deaths (Pune)

Other parameters were set as follows.

$S(0) = 740 \times 10^4$: Number of uninfected infected people as of 30th June, 2020

$E(0) = 10^4$: Number of people exposed but uninfected as of 30th June, 2020

$I(0) = 10305$: Number of infected people as of 30th June, 2020

$R(0) = 11270$: Number of recoverees as of 30th June, 2020

$N(0) = 7,411,100$: The population of Pune is approximately 7.4 million

$\lambda = \frac{80}{100}$: Infection rate

$\gamma = \frac{505}{1000}$: Recovery rate

$\varepsilon = \frac{34}{1000}$: Virus death rate

$\tau = 30$: 30 days to gain immunity (about 1 month required)

$\omega = 14$: A delay of 14 days before the disease develops (about 2 weeks of incubation period)

The time is t=0 indicates 30th June, 2020 (t=0), 8th October, 2020 (t = 100, Day 100 from the initial time), 16th January, 2021(t=200), and 26th April, 2021 (t=300).

The contrast table between t and month day is as follows.

x-axis	0	100	200	300
day/month	30 th June, 2020	8 th October, 2020	16 th January, 2021	26 th April, 2021

Findings and Discussion :

The number of reproductions in the case of Pune is $R_0 = 1.4842$ (WHO is $1.4 \leq R_0 \leq 2.5$).

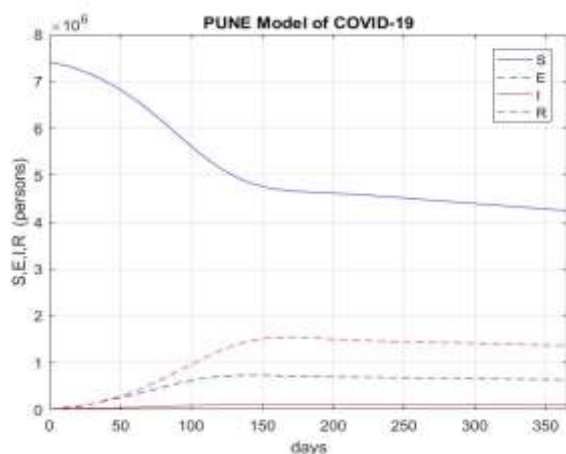


Figure 1

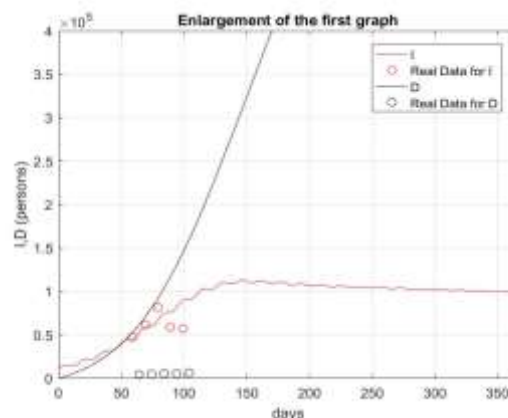


Figure 2

The main concern of the simulation is the transition of the number of infected people (I) and deaths (D), by expanding the vertical scale of Figure 1, Figure 2 makes them easier to see. The number of infected people is shown by the red solid line and the number of deaths is shown by the black solid line. In addition, the, \circ mark is the actual data, and it is well matched to the simulation at the initial state. The actual data was collected from the public website of Pune City.

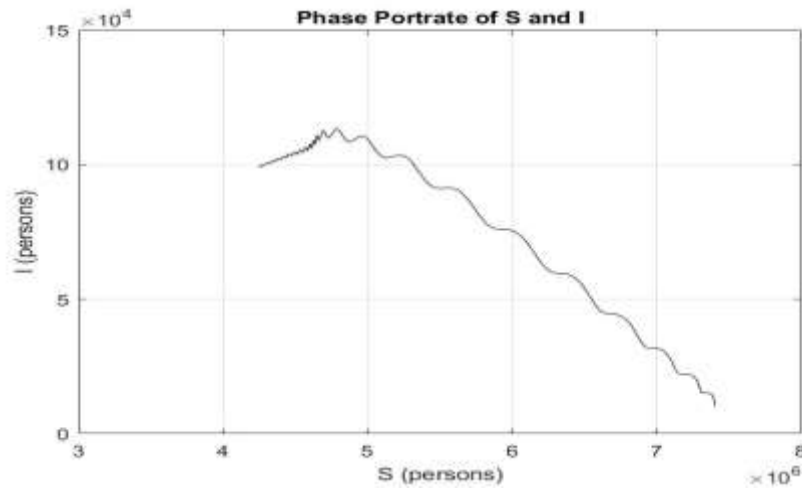


Figure 3

Figure 3: Phase diagram of sensitive population (S) and infected population (I)

This simulation is a calculation result of estimating each parameter of the model equation based on the number of infected people

Conclusion:

As the number of reproductions $R_0 = 1.4842$ is greater than 1 that means pandemic is not over yet. Also, the number of cases of covid-19 infections is also increasing. The death curve is not well fitted because the effect of vaccinations. After vaccination the death rate is decreased.

References:

Kermack, W.O., McKendrick, A.G., 1927. A contribution to the mathematical theory of epidemics. Proc. R. Soc. Lond. Ser. Contain. Pap. Math. Phys. Character 115, 700–721.

Ahmad, Z., Arif, M., Ali, F., Khan, I., Nisar, K.S., 2020. A report on COVID-19 epidemic in Pakistan using SEIR fractional model. Scientific Reports 10, 1–14.

Annas, S., Pratama, M.I., Rifandi, M., Sanusi, W., Side, S., 2020. Stability analysis and numerical simulation of SEIR model for pandemic COVID-19 spread in Indonesia. Chaos, Solitons & Fractals 139, 110072.

Atkeson, A., 2021. A parsimonious behavioral SEIR model of the 2020 COVID epidemic in the United States and the United Kingdom. National Bureau of Economic Research.

Bauch, C., Earn, D., 2003. Interepidemic intervals in forced and unforced SEIR models. Dynamical systems and their applications in biology 36, 33–44.

Berger, D.W., Herkenhoff, K.F., Mongey, S., 2020. An seir infectious disease model with testing and conditional quarantine. National Bureau of Economic Research.

- Biswas, M.H.A., Paiva, L.T., De Pinho, M., 2014. A SEIR model for control of infectious diseases with constraints. *Mathematical Biosciences & Engineering* 11, 761.
- De la Sen, M., Ibeas, A., Alonso-Quesada, S., 2012. On vaccination controls for the SEIR epidemic model. *Communications in Nonlinear Science and Numerical Simulation* 17, 2637–2658.
- Deguen, S., Thomas, G., Chau, N.P., 2000. Estimation of the contact rate in a seasonal SEIR model: application to chickenpox incidence in France. *Statistics in medicine* 19, 1207–1216.
- Diaz, P., Constantine, P., Kalmbach, K., Jones, E., Pankavich, S., 2018. A modified SEIR model for the spread of Ebola in Western Africa and metrics for resource allocation. *Applied mathematics and computation* 324, 141–155.
- Dubey, B., Patra, A., Srivastava, P., Dubey, U.S., 2013. Modeling and analysis of an SEIR model with different types of nonlinear treatment rates. *Journal of Biological Systems* 21, 1350023.
- Dukic, V., Lopes, H.F., Polson, N.G., 2012. Tracking epidemics with Google flu trends data and a state-space SEIR model. *Journal of the American Statistical Association* 107, 1410–1426.
- Efimov, D., Ushirobira, R., 2021. On an interval prediction of COVID-19 development based on a SEIR epidemic model. *Annual reviews in control*.
- Engbert, R., Rabe, M.M., Kliegl, R., Reich, S., 2021. Sequential data assimilation of the stochastic SEIR epidemic model for regional COVID-19 dynamics. *Bulletin of mathematical biology* 83, 1–16.
- Feng, Z., 2007. Final and peak epidemic sizes for SEIR models with quarantine and isolation. *Mathematical Biosciences & Engineering* 4, 675.
- Gao, S., Chen, L., Teng, Z., 2008a. Pulse vaccination of an SEIR epidemic model with time delay. *Nonlinear Analysis: Real World Applications* 9, 599–607.
- Gao, S., Teng, Z., Xie, D., 2008b. The effects of pulse vaccination on SEIR model with two time delays. *Applied Mathematics and Computation* 201, 282–292.
- Getz, W.M., Salter, R., Mgbara, W., 2019. Adequacy of SEIR models when epidemics have spatial structure: Ebola in Sierra Leone. *Philosophical Transactions of the Royal Society B* 374, 20180282.
- He, S., Peng, Y., Sun, K., 2020. SEIR modeling of the COVID-19 and its dynamics. *Nonlinear dynamics* 101, 1667–1680.
- Huo, H.-F., Yang, Q., Xiang, H., 2020. Dynamics of an edge-based SEIR model for sexually transmitted diseases. *Mathematical Biosciences and Engineering* 17, 669–699.
- Intissar, A., 2020. A mathematical study of a generalized SEIR model of COVID-19. *SciMedicine Journal* 2, 30–67.
- Kaddar, A., Abta, A., Alaoui, H.T., 2011. A comparison of delayed SIR and SEIR epidemic models. *Nonlinear Analysis: Modelling and Control* 16, 181–190.
- Korobeinikov, A., 2009. Global properties of SIR and SEIR epidemic models with multiple parallel infectious stages. *Bulletin of mathematical biology* 71, 75–83.
- Korobeinikov, A., 2004. Lyapunov functions and global properties for SEIR and SEIS epidemic models. *Mathematical medicine and biology: a journal of the IMA* 21, 75–83.

- Kounchev, O., Simeonov, G., Kuncheva, Z., 2020. The TVBG-SEIR spline model for analysis of COVID-19 spread, and a Tool for prediction scenarios. arXiv preprint arXiv:2004.11338.
- Lacitignola, D., Diele, F., 2021. Using awareness to Z-control a SEIR model with overexposure: insights on Covid-19 pandemic. *Chaos, Solitons & Fractals* 111063.
- Lekone, P.E., Finkenstädt, B.F., 2006. Statistical inference in a stochastic epidemic SEIR model with control intervention: Ebola as a case study. *Biometrics* 62, 1170–1177.
- Li, M.Y., Muldowney, J.S., 1995. Global stability for the SEIR model in epidemiology. *Mathematical biosciences* 125, 155–164.
- Li, M.Y., Wang, L., 2002. Global stability in some SEIR epidemic models, in: *Mathematical Approaches for Emerging and Reemerging Infectious Diseases: Models, Methods, and Theory*. Springer, pp. 295–311.
- López, L., Rodo, X., 2021. A modified SEIR model to predict the COVID-19 outbreak in Spain and Italy: simulating control scenarios and multi-scale epidemics. *Results in Physics* 21, 103746.
- Ma, J., Ma, Z., 2006. Epidemic threshold conditions for seasonally forced SEIR models. *Mathematical Biosciences & Engineering* 3, 161.
- Mojeeb, A., Adu, I.K., Yang, C., 2017. A simple seir mathematical model of malaria transmission. *Asian research journal of mathematics* 1–22.
- Pandey, G., Chaudhary, P., Gupta, R., Pal, S., 2020. SEIR and Regression Model based COVID-19 outbreak predictions in India. arXiv preprint arXiv:2004.00958.
- Peiliang, S., Li, K., 2020. An SEIR model for assessment of current COVID-19 pandemic situation in the UK. medRxiv.
- Piovella, N., 2020. Analytical solution of SEIR model describing the free spread of the COVID-19 pandemic. *Chaos, Solitons & Fractals* 140, 110243.
- Putra, Z.A., Abidin, S.A.Z., 2020. Application of SEIR model in COVID-19 and the effect of lockdown on reducing the number of active cases. *Indonesian Journal of Science and Technology*.
- Rachah, A., 2018. Analysis, simulation and optimal control of a SEIR model for Ebola virus with demographic effects. *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics* 67, 179–197.
- Rădulescu, A., Williams, C., Cavanagh, K., 2020. Management strategies in a SEIR-type model of COVID 19 community spread. *Scientific reports* 10, 1–16.
- Saito, M.M., Imoto, S., Yamaguchi, R., Sato, H., Nakada, H., Kami, M., Miyano, S., Higuchi, T., 2013. Extension and verification of the SEIR model on the 2009 influenza A (H1N1) pandemic in Japan. *Mathematical biosciences* 246, 47–54.
- Shah, N.H., Gupta, J., 2013. SEIR model and simulation for vector borne diseases.
- Syafruddin, S., Noorani, M., 2012. SEIR model for transmission of dengue fever in Selangor Malaysia. Presented at the *International Journal of Modern Physics Conference Series*, pp. 380–389.
- Tang, Z., Li, X., Li, H., 2020. Prediction of new coronavirus infection based on a modified SEIR model. medRxiv.

- Teles, P., 2020. A time-dependent SEIR model to analyse the evolution of the SARS-CoV-2 epidemic outbreak in Portugal. arXiv preprint arXiv:2004.04735.
- Yan, P., Liu, S., 2006. SEIR epidemic model with delay. *The ANZIAM Journal* 48, 119–134.
- Yang, Z., Zeng, Z., Wang, K., Wong, S.-S., Liang, W., Zanin, M., Liu, P., Cao, X., Gao, Z., Mai, Z., 2020. Modified SEIR and AI prediction of the epidemics trend of COVID-19 in China under public health interventions. *Journal of thoracic disease* 12, 165.
- Youssef, H.M., Alghamdi, N.A., Ezzat, M.A., El-Bary, A.A., Shawky, A.M., 2020. A modified SEIR model applied to the data of COVID-19 spread in Saudi Arabia. *AIP advances* 10, 125210.
- Zha, W., Pang, F., Zhou, N., Wu, B., Liu, Y., Du, Y., Hong, X., Lv, Y., 2020. Research about the optimal strategies for prevention and control of varicella outbreak in a school in a central city of China: Based on an SEIR dynamic model. *Epidemiology & Infection* 148.
- Zhang, J., Li, J., Ma, Z., 2006. Global dynamics of an SEIR epidemic model with immigration of different compartments. *Acta Mathematica Scientia* 26, 551–567.
- Zisad, S.N., Hossain, Mohammad Shahadat, Hossain, Mohammed Sazzad, Andersson, K., 2021. An integrated neural network and SEIR model to predict COVID-19. *Algorithms* 14, 94.
- Geng Hui, Xu Anding, Wang Xiaoyan, Daniel Zhang, Yin Xiaomei, Ma Mao, Lü Jun, 2020. Analysis of the role of current prevention and control measures in the epidemic of new coronavirus based on SEIR model. *Jinan University Academic Journal (Natural Science and Medicine Edition)*.
- Bajaj, N.S., Pardeshi, S.S., Patange, A.D., Kotecha, D., Mate, K.K., 2020. Statistical analysis of national & municipal corporation level database of COVID-19 cases In India. medRxiv.
- Bhapkar, H., Mahalle, P.N., Dhotre, P.S., 2020. Virus graph and COVID-19 pandemic: a graph theory approach, in: *Big Data Analytics and Artificial Intelligence against COVID-19: Innovation Vision and Approach*. Springer, pp. 15–34.
- Darapaneni, N., Nikam, D., Lomate, A., Kherde, V., Katdare, S., Paduri, A.R., Rao, K., Shukla, A., 2020. Coronavirus Outburst Prediction in India using SEIRD, Logistic Regression and ARIMA Model. Presented at the 2020 11th IEEE Annual Ubiquitous Computing, Electronics & Mobile Communication Conference (UEMCON), IEEE, pp. 0649–0655.
- Dixit, R., Panda, D.S., Panda, S.S., 2021. An Advanced Susceptible-Exposed-Infectious-Recovered model for quantitative analysis of COVID-19. *Sādhanā* 46, 1–10.
- Mahalle, P.N., Sable, N.P., Mahalle, N.P., Shinde, G.R., 2020. Data analytics: COVID-19 prediction using multimodal data, in: *Intelligent Systems and Methods to Combat Covid-19*. Springer, pp. 1–10.
- Prasad, J., 2020. A data first approach to modelling Covid-19. medRxiv.
- Pujari, B.S., Shekatkar, S.M., 2020. Multi-city modeling of epidemics using spatial networks: Application to 2019-nCov (COVID-19) coronavirus in India. Medrxiv.
- Rajesh, A., Pai, H., Roy, V., Samanta, S., Ghosh, S., 2020. CoVID-19 prediction for India from the existing data and SIR (D) model study. medRxiv.
- Rapolu, T., Nutakki, B., Sobha Rani, T., Durga Bhavani, S., 2022. A Time-Dependent SEIRD Model for Forecasting the Transmission Dynamics in Infectious Diseases: COVID-19 a Case Study. Presented

at the Proceedings of International Conference on Data Science and Applications, Springer, pp. 423–437.

Sahasranaman, A., Jensen, H.J., 2021. Spread of COVID-19 in urban neighbourhoods and slums of the developing world. *J. R. Soc. Interface* 18, 20200599.

Sahoo, B.K., Sapra, B.K., 2020. A data driven epidemic model to analyse the lockdown effect and predict the course of COVID-19 progress in India. *Chaos Solitons Fractals* 139, 110034.

Shaikh, A.S., Jadhav, V.S., Timol, M.G., Nisar, K.S., Khan, I., 2020. Analysis of the COVID-19 pandemic spreading in India by an epidemiological model and fractional differential operator.

Wagh, C.S., Mahalle, P.N., Wagh, S.J., 2020. Epidemic peak for COVID-19 in India, 2020.

Welling, A., Patel, A., Kulkarni, P., Vaidya, V.G., 2021. A Novel Systems Model to Simulate the Dynamics for the Analytical Study of the Proliferation of COVID-19.

Yadav, D.K., Shukla, S., Yadav, S., 2020. Analyzing the Current Status of India in Global Scenario with Reference to COVID-19 Pandemic.

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