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SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS BY DOUBLE LAPLACE – UPADHYAYA TRANSFORM

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Abstract:

In this paper we are going to solve numerous partial differential equations by introducing a new double integral transform known as Laplace – Upadhyaya Transform (DLUT). The significance of this research is that all the other double Laplace – any transform, merely becomes particular case of this newly introduced DLUT. In this paper we will prove few theorems and properties of DLUT which in return will be useful to solve PDE's.

Keywords: *Double Laplace – Upadhyaya Transform, Single Laplace Transform, Single Upadhyaya Transform, Partial Differential Equations.*

Introduction:

Double Integral Transforms has become an active area for new researchers in the last few decades. It is a powerful technique used to solve partial differential equations as well as fractional differential equations of various kind. Some of the work using double integral transform can be referred from [1-6]. In the paper [2] solution of partial differential equations is obtained by using double Laplace – Sumudu transform. Recently, Upadhyaya, L. M. [7] introduced a new transform known as Upadhyaya transform which we have used in our research. In this paper we use double Laplace – Upadhyaya Transform to solve partial differential equations. Beauty of this research is, results obtained to solve partial differential equations by any double Laplace – some transform, will merely become particular cases of our results.

Definition 1:

The Laplace Transform of the continuous function $h(x)$ is defined by,

$$L[h(x)] = \int_0^\infty e^{-\rho x} h(x)\, dx = H(\rho)$$

Definition 2:

The Upadhyaya Transform of the continuous function $f(t)$ is defined by [7],

$$\mathcal{U}[f(t)] = \lambda_1 \int_0^\infty e_2^{-\lambda_2 t} f(\lambda_3 t) dt = F(\lambda_1, \lambda_2, \lambda_3).$$

Definition 3:

The double Laplace – Upadhyaya transform of the function $h(x,t)$ of two variables $x > 0$ and $t > 0$ is denoted by,

$L_x\mathcal{U}_t[h(x,t)] = \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t}\, h(x,\lambda_3 t)\, dxdt = H(\rho,\lambda_1,\lambda_2,\lambda_3).$

Note that,

$$L_x\mathcal{U}_t[\alpha_1 h(x,t)+\alpha_2 g(x,t)] = \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t}[\,\alpha_1 h(x,\lambda_3 t)+\alpha_2 g(x,\lambda_3 t)]\, dxdt$$

$$= \alpha_1\lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t}\, h(x,\lambda_3 t) dxdt + \alpha_2\lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t}[\, g(x,\lambda_3 t)]\, dxdt$$

$= \alpha_1 L_x\mathcal{U}_t[h(x,t)] + \alpha_2 L_x\mathcal{U}_t[g(x,t)]$, where α_1, α_2 are constants.

Definition 4:

The inverse Laplace – Upadhyaya transform $L_x^{-1}\mathcal{U}_t^{-1}[H(\rho,\lambda_1,\lambda_2,\lambda_3)] = h(x,t)$ is defined by,

$L_x^{-1}\mathcal{U}_t^{-1}[H(\rho,\lambda_1,\lambda_2,\lambda_3)] = \left(\frac{1}{2\pi i}\right) \int_{\gamma-i\infty}^{\gamma+i\infty} e^{\rho x} d\rho \left(\frac{1}{2\pi i}\right) \int_{\omega-i\infty}^{\omega+i\infty} \lambda_1 e^{\lambda_2 t} H(\rho,\lambda_1,\lambda_2,\lambda_3) d\lambda_2.$

Double Laplace – Upadhyaya transform of some basic functions:

(1) $h(x,t) = 1,\ x > 0,\ t > 0$

$$L_x\mathcal{U}_t[1] = \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t}\ dxdt$$

$$= \lambda_1 \left[\int_0^\infty e^{-\rho x}\, dx \ \int_0^\infty e^{-\lambda_2 t}\, dt\right]$$

$$= \frac{\lambda_1}{\rho\lambda_2}$$

(2) $h(x,t) = x^a\, t^b$

$$L_x\mathcal{U}_t[x^a t^b] = \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t}\, x^a\, (\lambda_3 t)^b\, dxdt$$

$$= \lambda_1\lambda_3^b \left[\int_0^\infty e^{-\rho x} x^c\, dx \ \int_0^\infty e^{-\lambda_2 t}\, t^b dt\right]$$

$= \frac{\lambda_1\lambda_3^b\Gamma(a+1)}{\rho^{a+1}} \frac{\Gamma(b+1)}{\lambda_2^{b+1}}$, $Re[a] > -1, Re[b] > -1.$

If a and b are positive integers, then

$L_x\mathcal{U}_t[x^a t^b] = \frac{\lambda_1\lambda_3^b}{\lambda_2^{b+1}} \frac{a!b!}{\rho^{a+1}}$

(3) $h(x,t) = e^{ax+bt}$

$$L_x\mathcal{U}_t[e^{ax+bt}] = \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t}\, e^{ax+b\lambda_3 t}\, dxdt$$

$$= \lambda_1 \left[\int_0^\infty e^{-\rho x+ax}\ dx \ \int_0^\infty e^{-\lambda_2 t+b\lambda_3 t}\, dt\right]$$

$= \frac{\lambda_1}{(a-\rho)(b\lambda_3-\lambda_2)}$

Similarly,

$$L_x\mathcal{U}_t\left[e^{i(ax+bt)}\right] = \frac{\lambda_1}{(ia-\rho)(ib\lambda_3-\lambda_2)}$$

$$= \frac{\lambda_1[(\rho\lambda_2 - ab\lambda_3) + (\rho b\lambda_3 + a\lambda_2)]}{(\rho^2+a^2)(\lambda_2^2+b^2\lambda_3^2)}$$

By using above equation we get,

$$L_x\mathcal{U}_t[\sin(ax+bt)] = \frac{\lambda_1(\rho b\lambda_3 + a\lambda_2)}{(\rho^2+a^2)(\lambda_2^2+b^2\lambda_3^2)}$$

$$L_x\mathcal{U}_t[\cos(ax+bt)] = \frac{\lambda_1(\rho\lambda_2 - ab\lambda_3)}{(\rho^2+a^2)(\lambda_2^2+b^2\lambda_3^2)}$$

(4) $h(x,t) = \sinh(cx+dt)$ *or* $\cosh(cx+dt)$

We know that $\sinh y = \frac{(e^y - e^{-y})}{2}$ and $\cosh y = \frac{(e^y + e^{-y})}{2}$.

Thus,

$$L_x\mathcal{U}_t[\sinh(cx+dt)] = \lambda_1\left[\frac{a\lambda_2 + b\lambda_3\rho}{(a^2-\rho^2)(b^2\lambda_3^2-\lambda_2^2)}\right]$$

$$L_x\mathcal{U}_t[\cosh(cx+dt)] = \lambda_1\left[\frac{ab\lambda_3 + \lambda_2\rho}{(a^2-\rho^2)(b^2\lambda_3^2-\lambda_2^2)}\right]$$

(5) $h(x,t) = f(x)g(t)$, then

$$L_x\mathcal{U}_t[h(x,t)] = \lambda_1\int_0^\infty\int_0^\infty e^{-\rho x-\lambda_2 t} f(x)g(\lambda_3 t)\,dxdt$$

$$= \int_o^\infty e^{-\rho x} f(x)dx\ \lambda_1\int_0^\infty e^{-\lambda_2 t} g(\lambda_3 t)dt$$

$$= L_x[f(x)]\mathcal{U}_t[g(t)]$$

Existence condition for the double Laplace – Upadhyaya transform:

If $h(x,t)$ is of exponential order a and b as $x\to\infty,\ t\to\infty$, if there exist a positive constant K such that $\forall\, x > X, t > T$

$$|h(x,t)| = Ke^{ax+bt},$$

And $\lim_{x\to\infty,t\to\infty} e^{-\rho x-\lambda_2 t}\ |h(x,t)| = 0, \rho > a, \lambda_2 > b.$

Thus the function $h(x,t)$ is said to be of exponential order.

Theorem 1:

If a function $h(x,t)$ is a continuous function in every finite interval $(0,X)$ and $(0,T)$ of exponential order e^{ax+bt}, then the double Laplace – Upadhyaya transform of $h(x,t)$ exists for all ρ and λ_2 provided $Re[\rho] > a$ and $Re[\lambda_2] > b$.

Proof:

From the definition (3) we have,

$$|H(\rho,\lambda_1,\lambda_2,\lambda_3)| = \left|\lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t} h(x,\lambda_3 t)\, dxdt\right|$$

$$\leq K \int_0^\infty e^{-(\rho-a)x}\, dx \int_0^\infty \lambda_1 e^{-(\lambda_2-\lambda_3 b)t}\, dt$$

$$= \frac{K}{(\rho-a)(1-\frac{b\lambda_3}{\lambda_2})}, Re[\rho] > a, Re[\lambda_2] > b$$

$$\therefore \lim_{x,t\to\infty} |H(\rho,\lambda_1,\lambda_2,\lambda_3)| = 0$$

Basic properties of double Laplace – Upadhyaya transform:

Shifting Property:

If $L_x\mathcal{U}_t[h(x,t)] = H(\rho,\lambda_1,\lambda_2,\lambda_3)$ then $L_x\mathcal{U}_t[e^{ax+bt} h(x,t)] = H(\rho-a,\lambda_1,\lambda_2-b,\lambda_3)$.

Proof is left as an exercise to the reader.

Properties of derivatives:

If $L_x\mathcal{U}_t[h(x,t)] = H(\rho,\lambda_1,\lambda_2,\lambda_3)$ then

i. $L_x\mathcal{U}_t\left[\frac{\partial h(x,t)}{\partial x}\right] = \rho H(\rho,\lambda_1,\lambda_2,\lambda_3) - \mathcal{U}[h(0,t)]$

ii. $L_x\mathcal{U}_t\left[\frac{\partial h(x,t)}{\partial t}\right] = \lambda_2 H(\rho,\lambda_1,\lambda_2,\lambda_3) - \lambda_1 L[h(x,0)]$

iii. $L_x\mathcal{U}_t\left[\frac{\partial^2 h(x,t)}{\partial x^2}\right] = \rho^2 H(\rho,\lambda_1,\lambda_2,\lambda_3) - \rho\mathcal{U}[h(0,t)] - \mathcal{U}[h_x(0,t)]$

iv. $L_x\mathcal{U}_t\left[\frac{\partial^2 h(x,t)}{\partial t^2}\right] = \lambda_2^2 H(\rho,\lambda_1,\lambda_2,\lambda_3) - \lambda_1 L[h_t(x,0)] - \lambda_1\lambda_2 L[h(x,0)]$

v. $L_x\mathcal{U}_t\left[\frac{\partial^2 h(x,t)}{\partial x\partial t}\right] = \lambda_2\rho H(\rho,\lambda_1,\lambda_2,\lambda_3) - \lambda_1\rho L[h(x,0)] - \mathcal{U}[h_t(0,t)]$

Proof:

i. $L_x\mathcal{U}_t\left[\frac{\partial h(x,t)}{\partial x}\right] = \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t} \frac{\partial h(x,t)}{\partial t}\, dxdt$

$$= \lambda_1 \int_0^\infty e^{-\lambda_2 t}\, dt \int_0^\infty \frac{e^{-\rho x}\partial h(x,\lambda_3 t)}{\partial x}\, dx$$

$$= \lambda_1 \int_0^\infty e^{-\lambda_2 t}\, dt \left\{-h(0,\lambda_3 t) + \rho \int_0^\infty e^{-\rho x} h(x,\lambda_3 t)\ dx\right\}$$

$$= \rho\lambda_1 \int_0^\infty \int_0^\infty e^{-\lambda_2 t-\rho x}\, h(x,\lambda_3 t) dxdt - \lambda_1 \int_0^\infty e^{-\lambda_2 t} h(0,\lambda_3 t) dt$$

$$= \rho H(\rho,\lambda_1,\lambda_2,\lambda_3) - \mathcal{U}[h(0,\lambda_3 t)]$$

Note: Proofs of ii. to v. is left as an exercise to the reader.

Theorem 2:

If $H(\rho,\lambda_1,\lambda_2,\lambda_3) = L_x\mathcal{U}_t[\, h(x,t)]$, then

$$L_x\mathcal{U}_t[h(x-\delta,t-\epsilon\,)H(x-\delta,t-\epsilon)] = e^{-\rho\delta-\frac{\epsilon\lambda_2}{\lambda_3}} H(\rho,\lambda_1,\lambda_2,\lambda_3)$$

Where $H(x,t)$ is a Heaviside unit step function given by,

$$H(x-\delta,t-\epsilon) = \begin{cases} 1, & x>\delta, t>\epsilon \\ 0, & otherwise \end{cases}$$

Proof:

$$L_x\mathcal{U}_t\left[h(x-\delta,t-\epsilon\,)H(x-\delta,t-\epsilon) = \lambda_1 \int_0^\infty \int_0^\infty \left[e^{-\rho x-\lambda_2 t} h(x-\delta,\lambda_3 t-\epsilon)H(x-\delta,\lambda_3 t-\epsilon)\right] dxdt\right]$$

$$= \lambda_3 \int_\delta^\infty \int_{\frac{\epsilon}{\lambda_3}}^\infty e^{-\rho x-\lambda_2 t} h(x-\delta,\lambda_3 t-\epsilon)\ dxdt$$

$$put\ x-\delta = f, \qquad \lambda_3 t-\epsilon = \lambda_3 g$$

$$dx = df, \qquad \lambda_3 dt = dg$$

$$= \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho(f+\delta)-\lambda_2\left(\frac{\epsilon+g\lambda_3}{\lambda_3}\right)} h(f,\lambda_3 g)\ dfdg$$

$$= \lambda_1 e^{\rho\delta-\frac{\lambda_2\epsilon}{\lambda_3}} \int_0^\infty \int_0^\infty e^{-\rho f-\lambda_2 g} h(f,\lambda_3 g)\ dfdg$$

$$= e^{-\rho\delta-\frac{\epsilon\lambda_2}{\lambda_3}} H(\rho,\lambda_1,\lambda_2,\lambda_3)$$

Theorem 3:

If the double Laplace – Upadhyaya transform of $h(x,t)$ exists, where $h(x,t)$ is a periodic function of periods a and b such that $h(x+a,t+b) = h(x,t)\ \forall\ x,t$ then,

$$H(\rho,\lambda_1,\lambda_2,\lambda_3) = \left[1-e^{-\rho a-\frac{\lambda_2 b}{\lambda_3}}\right]^{-1} \lambda_1 \int_0^a \int_0^{\frac{b}{\lambda_3}} e^{-\rho x-\lambda_2 t}\, h(x,\lambda_3 t) dxdt\,.$$

Proof:

$$H(\rho,\lambda_1,\lambda_2,\lambda_3) = \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t}\, h(x,\lambda_3 t)dxdt$$

$$= \lambda_1 \int_0^a \int_0^{\frac{b}{\lambda_3}} e^{-\rho x-\lambda_2 t} h(x,\lambda_3 t)dxdt + \lambda_1 \int_a^\infty \int_{\frac{b}{\lambda_3}}^\infty e^{-\rho x-\lambda_2 t} h(x,\lambda_3 t)dxdt$$

Put $x - a = f, \lambda_3 t - b = \lambda_3 g$ in the second term

$$= \lambda_1 \int_0^a \int_0^{\frac{b}{\lambda_3}} e^{-\rho x-\lambda_2 t} h(x,\lambda_3 t)dxdt + e^{-\rho a-\frac{\lambda_2 b}{\lambda_3}} \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho f-\lambda_2 g} h(f+a,\lambda_3 g+b)dfdg$$

$$= \lambda_1 \int_0^a \int_0^{\frac{b}{\lambda_3}} e^{-\rho x-\lambda_2 t} h(x,\lambda_3 t)dxdt + e^{-\rho a-\frac{\lambda_2 b}{\lambda_3}} \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho f-\lambda_2 g} h(f,\lambda_3 g)dfdg$$

$$= \lambda_1 \int_0^a \int_0^{\frac{b}{\lambda_3}} e^{-\rho x-\lambda_2 t} h(x,\lambda_3 t)dxdt + e^{-\rho a-\frac{\lambda_2 b}{\lambda_3}} H(\rho,\lambda_1,\lambda_2,\lambda_3)$$

$$\therefore\; H(\rho,\lambda_1,\lambda_2,\lambda_3) = \left[1 - e^{-\rho a-\frac{\lambda_2 b}{\lambda_3}}\right]^{-1} \lambda_1 \int_0^a \int_0^{\frac{b}{\lambda_3}} e^{-\rho x-\lambda_2 t}\, h(x,\lambda_3 t)dxdt.$$

Convolution theorem of double Laplace – Upadhyaya transform:

Definition 5:

The convolution of $h(x,t)$ and $g(x,t)$ is denoted by $(h ** g)(x,t)$ and is defined by,

$$(h ** g)(x,t) = \int_0^x \int_0^t h(x-\delta, t-\epsilon) g(\delta,\epsilon)\, d\delta\, d\epsilon$$

Theorem 4 (Convolution theorem):

If $H(\rho,\lambda_1,\lambda_2,\lambda_3) = L_x \mathcal{U}_t[\,h(x,t)]$, and $G(\rho,\lambda_1,\lambda_2,\lambda_3) = L_x \mathcal{U}_t[\,g(x,t)]$, then

$$L_x \mathcal{U}_t[(h ** g)(x,t)] = \frac{\lambda_3}{\lambda_1} H(\rho,\lambda_1,\lambda_2,\lambda_3) G(\rho,\lambda_1,\lambda_2,\lambda_3)$$

Proof:

$$L_x \mathcal{U}_t[(h ** g)(x,t)] = \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t} (h ** g)(x,t)dxdt$$

$$= \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t} \left[\int_0^x \int_0^t h(x-\delta,t-\epsilon) g(\delta,\epsilon)\, d\delta\, d\epsilon\right] dxdt$$

Using Heaviside unit step function,

$$= \lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t} \left[\int_0^x \int_0^t h(x-\delta,t-\epsilon) H(x-\delta,t-\epsilon) g(\delta,\epsilon)\, d\delta\, d\epsilon\right] dxdt$$

$$= \int_0^\infty \int_0^\infty g(\delta,\epsilon)\, d\delta\, d\epsilon \left\{\lambda_1 \int_0^\infty \int_0^\infty e^{-\rho x-\lambda_2 t} h(x-\delta,t-\epsilon)H(x-\delta,t-\epsilon)\, dxdt\right\}$$
$$= \int_0^\infty \int_0^\infty g(\delta,\epsilon)\, d\delta\, d\epsilon \left\{e^{-\rho\delta-\frac{\epsilon\lambda_2}{\lambda_3}} H(\rho,\lambda_1,\lambda_2,\lambda_3)\right\}$$
$$= H(\rho,\lambda_1,\lambda_2,\lambda_3)\int_0^\infty \int_0^\infty e^{-\rho\delta-\frac{\epsilon\lambda_2}{\lambda_3}} g(\delta,\epsilon)\; d\delta\, d\epsilon = \frac{\lambda_3}{\lambda_1} H(\rho,\lambda_1,\lambda_2,\lambda_3)$$

Application of DLUT to solve partial differential equations:

Following type of partial differential equations can be solved using DLUT,

$$AU_{xx} + BU_{tt} + CU_x + DU_t + EU(x,t) = g(x,t) \quad (1)$$

With initial conditions:

$$U(x,0) = h_1(x),\; U_t(x,0) = h_2(x) \quad (2)$$

And the boundary conditions:

$$U(0,t) = h_3(t),\; U_x(0,t) = h_4(t), \quad (3)$$

Where A, B, C, D and E are constants.

Using property of DLUT, single Laplace transform and single Upadhyaya transform to (1), (2) and (3) respectively we get,

$$U(\rho,\lambda_1,\lambda_2,\lambda_3) = \left[\frac{1}{A\rho^2 + B\lambda_2^2 + C\rho + D\lambda_2 + E}[Ah_4(\lambda_1,\lambda_2,\lambda_3) + A\rho h_3(\lambda_1,\lambda_2,\lambda_3) + B\lambda_1 h_2(\rho) + B\lambda_1\lambda_2 h_1(\rho) + Ch_3(\lambda_1,\lambda_2,\lambda_3) + D\lambda_1 h_1(\rho) + G(\rho,\lambda_1,\lambda_2,\lambda_3)]\right]$$

$$\therefore U(x,t) = L_x^{-1}\mathcal{U}_t^{-1}\left[\frac{1}{A\rho^2+B\lambda_2^2+C\rho+D\lambda_2+E}[Ah_4(\lambda_1,\lambda_2,\lambda_3) + A\rho h_3(\lambda_1,\lambda_2,\lambda_3) + B\lambda_1 h_2(\rho) + B\lambda_1\lambda_2 h_1(\rho) + Ch_3(\lambda_1,\lambda_2,\lambda_3) + D\lambda_1 h_1(\rho) + G(\rho,\lambda_1,\lambda_2,\lambda_3)]\right] \quad (4)$$

Illustrated Examples:

Example 1

Consider the homogenous wave equation [3] $U_{tt} = U_{xx}$,

With initial conditions,

$$U(x,0) = sinx, U_t(x,0) = 2$$
$$U(0,t) = 2t, U_x(0,t) = cost$$

put $h_1(\rho) = \frac{1}{\rho^2+1}, h_2(\rho) = \frac{2}{\rho},\; h_3(\lambda_1,\lambda_2,\lambda_3) = \frac{2\lambda_1\lambda_3}{\lambda_2^2},\; h_4(\lambda_1,\lambda_2,\lambda_3) = \frac{\lambda_1\lambda_2}{\lambda_2^2+\lambda_3^2}$ in (4) to get the solution,

$$U(x,t) = L_x^{-1}\mathcal{U}_t^{-1}\left[\frac{1}{(\lambda_2^2-\rho^2)}\left[\frac{2\lambda_1}{\rho} - \frac{2\rho\lambda_1\lambda_3}{\lambda_2^2} + \frac{\lambda_1\lambda_2}{(\rho^2+1)} - \frac{\lambda_1\lambda_2}{(\lambda_2^2+\lambda_3^2)}\right]\right] = 2t + sinx\; cost.$$

Example 2

Consider the homogenous telegraph equation [3],

$U_{xx} = U_{tt} + U_t - U$

With the conditions,

$$U(x,0) = e^x = h_1(x), \qquad U_t(x,0) = -2e^x = h_2(x)$$
$$U(0,t) = e^{-2t} = h_3(t), \qquad U_x(0,t) = e^{-2t} = h_4(t)$$

Put, $h_1(\rho) = \frac{1}{\rho-1}, h_2(\rho) = \frac{-2}{\rho-1},\ h_3(\lambda_1,\lambda_2,\lambda_3) = \ h_4(\lambda_1,\lambda_2,\lambda_3) = \frac{\lambda_1}{\lambda_2+2\lambda_3}$ in (4) to get the solution,

$$U(x,t) = L_x^{-1}\mathcal{U}_t^{-1}\left[\frac{\lambda_1}{\rho^2 - \lambda_2^2 - \lambda_2 + 1}\left[\frac{1+\rho}{\lambda_2 + 2\lambda_3} + \frac{1-\lambda_2}{\rho - 1}\right]\right] = e^{x-2t}$$

Conclusion:

In this paper we have introduced a new double transform known as double Laplace – Upadhyaya Transform. We also proved some elementary properties and basic theorems for DLUT. We further went on to solve partial differential equations of the type (1).

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