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SOLUTION OF POPULATION GROWTH AND RADIOACTIVE DECAY PROBLEMS BY YANG TRANSFORM METHOD

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Abstract:

"Yang Transform" is introduced by Xiao-Jun Yang[1] for solving Steady Heat-Transfer Problem in 2016. In this paper researcher used "Yang Transform" to find solution of linear differential equation satisfied by Population Growth and Radioactive Decay Problems.

Keywords: *Integral Transform, Yang Transform, Linear Differential Equation, Population Growth Model, Radioactive Decay Problem.*

1.INTRODUCTION

Differential equations are one of the most fascinating fields of mathematics having great practical importance [2]. Differential equations play fundamental role in Applied Mathematics, Physics, Chemistry, Biology, Engineering, Economics and so on. To solve the differential equations an integral transform was extensively used like Laplace Transform, fourier Transfom, Mellins Transforms, Sumudu Transform, Elzaki Transform, Aboodh Transform. The importance of an integral transforms is that they provide powerful operational methods for solving initial value and boundary value problems for linear differential and integral equations[3-6].

The main objective of this paper is to use Yang transform for finding solution of population growth and radioactive decay problems which is given below.

Let P is the size of a population of at any given time t and r is the population growth rate, then the rate of change of population with time is given by linear differential equation

$$\frac{dP}{dt} = rP \qquad (1)$$

with initial condition $P(0) = P_0$. A differential equation in equation (1) is known as Malthusian Law of Population.

Similarly, if N is the number of radioactive nuclei present in the sample at any time t, then the decay of a radioactive element can be described by

$$\frac{dN}{dt} = -\lambda N \qquad (2)$$

where λ is decay constant and $N(0) = N_0$ is number of radioactive nuclei originally present.

2 Preliminaries

2.1 YANG TRANSFORM

Definition: A new integral transform called Yang Transform of the function f(t) is denoted by Y{f(t)} or T(u) and is defined as

$$Y\{f(t)\} = T(u) = \int_0^\infty e^{\frac{-t}{u}} f(t)dt, t > 0 \qquad (3)$$

Provided the integral exists for some u, where $u \in (-t_1, t_2)$

2.2 Yang Transforms of Some Standard Functions

Sr. No.	f(t)	Y{f(t)} = T(u)
(i)	1	u
(ii)	t	u^2
(iii)	t^n	$n!\,u^{n+1}$
(iv)	e^{at}	$\frac{u}{1-au}$
(v)	e^{-at}	$\frac{u}{1+au}$
(vi)	sinat	$\frac{au^2}{1+a^2u^2}$
(vii)	cosat	$\frac{u}{1+a^2u^2}$
(viii)	sinhat	$\frac{au^2}{1-a^2u^2}$
(ix)	coshat	$\frac{u}{1-a^2u^2}$

2.3 Linearity Property for Yang Transform

If $Y\{f(t)\} = T_1(u)$ and $Y\{g(t)\} = T_2(u)$, then $Y\{af(t) + bg(t)\} = aT_1(u) + bT_2(u)$

2.4 Yang Transform of Derivatives

If Y{f(t)} = T(u), then

(i) $Y\{f'(t)\} = \frac{T(u)}{u} - f(0)$

(ii) $Y\{f''(t)\} = \frac{T(u)}{u^2} - \frac{f(0)}{u} - f'(0)$

(iii) $Y\{f^n(t)\} = \frac{T(u)}{u^n} - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{u^{n-k-1}} \; \forall\, n = 1,2,\dots,n$

2.5 Inverse Yang Transform

If $Y\{f(t)\} = T(u)$, then f(t) is called the inverse Yang Transform of T(u) and

$f(t) = Y^{-1}\{T(u)\}$.

2.6 Linearity Property for Inverse Yang Transform

If $Y^{-1}\{T_1(u)\} = f_1(t)$ *and* $Y^{-1}\{T_2(u)\} = f_2(t)$ *then,*

$Y^{-1}(aT_1(u) + bT_2(u)) = af_1(t) + bf_2(t)$

2.7 Inverse Yang Transforms of Some Standard Functions

Sr. No.	T(u)	$f(t) = Y^{-1}\{T(u)\}$
(i)	u	1
(ii)	u^2	t
(iii)	$n!\,u^{n+1}$	t^n
(iv)	$\frac{u}{1-au}$	e^{at}
(v)	$\frac{u}{1+au}$	e^{-at}
(vi)	$\frac{au^2}{1+a^2u^2}$	sinat
(vii)	$\frac{u}{1+a^2u^2}$	cosat
(viii)	$\frac{au^2}{1-a^2u^2}$	sinhat
(ix)	$\frac{u}{1-a^2u^2}$	coshat

3. APPLICATION OF YANG TRANSFORM

3.1 Solution of Population Growth Problem

Population growth problem is given by equation (1),

$$\frac{dP}{dt} = rP$$

Taking Yang Transform on both sides, we get

$$Y\{\frac{dP}{dt}\} = Y\{rP\}$$

$$\therefore \frac{T(u)}{u} - P(0) = rT(u)$$

$$\Rightarrow \frac{T(u)}{u} - rT(u) = P(0)$$

But, $P(0) = P_0$

$$\Rightarrow (\frac{1}{u} - r)T(u) = P_0$$

$$\Rightarrow T(u) = P_0 \frac{u}{1-ur} \qquad (4)$$

Now taking Inverse Yang Transform of equation (4), we get

$$Y^{-1}(T(u)) = Y^{-1}\left(P_0 \frac{u}{1-ur}\right)$$

$$\therefore P(t) = P_0 e^{rt} \qquad (5)$$

Equation (5) is the exponential population growth function.

3.2 Solution of Radioactive Decay Problem:

Radioactive decay problem is given by equation (2)

$$\frac{dN}{dt} = -\lambda N$$

Taking Yang Transform on both sides, we get

$$Y\{\frac{dN}{dt}\} = Y\{-\lambda N\}$$

$$\therefore \frac{T(u)}{u} - N(0) = -\lambda T(u)$$

$$\Rightarrow \frac{T(u)}{u} + \lambda T(u) = N(0)$$

But, $N(0) = N_0$

$$\Rightarrow (\frac{1}{u} + \lambda)T(u) = N_0$$

$$\Rightarrow T(u) = N_0 \frac{u}{1 + u\lambda} \qquad (6)$$

Now taking Inverse Yang Transform of equation (6), we get

$$Y^{-1}(T(u)) = Y^{-1}\left(N\frac{u}{1 + u\lambda}\right)$$

$$\therefore N(t) = N_0 e^{-\lambda t} \qquad (7)$$

Equation (7) is the exponential radioactive decay function.

4. PROBLEMS [7-8]

Example (4.1) If a population of a country doubles in 50 years, in how many years will it treble, assuming that the rate of increase is proportional to the number of inhabitants?

Solution:

Let P be the population at any time t, and P_0 is original population at t = 0. The rate of increase is proportional to the population,

$$\frac{dP}{dt} = rP$$

The solution is given by equation (5)

$$P(t) = P_0 e^{rt}$$

∵ Population of a country doubles in 50 years

i.e when t = 50, P(50) = $2P_0$,

But,

$$P(50) = P_0 e^{50r}$$

$$\therefore 2P_0 = P_0 e^{50r}$$

$$\therefore 2 = e^{50r}$$

Now taking logarithm on both sides, we get

$$r = \frac{\log 2}{50}$$

The exponential growth function is given by

$$\therefore \mathrm{P(t)} = \mathrm{P_0 e}^{\left(\frac{log2}{50}\right)\mathrm{t}} \qquad (8)$$

The value of t when population has trebled is obtained from (8)

$$3\mathrm{P_0} = \mathrm{P_0 e}^{\left(\frac{log2}{50}\right)\mathrm{t}}$$

$$\Rightarrow 3 = \mathrm{e}^{\left(\frac{log2}{50}\right)\mathrm{t}}$$

$$\Rightarrow \mathrm{t} = 50\left(\frac{log3}{log2}\right)$$

$$\Rightarrow \mathrm{t} = 79.248125035 \text{ years}$$

The population of country will be trebled in 79.248125035 years.

Example (2) Radium decomposes at a rate proportional to the amount present. If 5% of the original amount disappears in 50 years, how much will remain after 100 years?

Solution: let N be the amount of Radium at any time t,

$$\frac{\mathrm{dN}}{\mathrm{dt}} = -\lambda \mathrm{N}$$

Let $N(0) = N_0$, when t = 0. The solution is given by equation (7)

$$\mathrm{N(t)} = N_0 \mathrm{e}^{-\lambda \mathrm{t}}$$

Given that 5% of original amount disappears in 50 years

$$\therefore \mathrm{N(50)} = 0.95\mathrm{N_0}$$

But,

$$\mathrm{N(50)} = \mathrm{N_0 e}^{-50\lambda}$$

$$\Rightarrow 0.95N_0 = \mathrm{N_0 e}^{-50\lambda}$$

$$\Rightarrow 0.95 = \mathrm{e}^{-50\lambda}$$

Taking logarithm on both sides, we get

$$\lambda = -\frac{log0.95}{50}$$

$$\therefore \mathrm{N(t)} = \mathrm{N_0 e}^{\left(\frac{log0.95}{50}\right)\mathrm{t}} \qquad (9)$$

Amount after 100 years will be

$$\mathrm{N(100)} = \mathrm{N_0 e}^{\left(\frac{log0.95}{50}\right)100}$$

$$\therefore \mathrm{N(100)} = \mathrm{N_0 e}^{2\log 0.95}$$

$$\therefore \mathrm{N(100)} = 0.9025\mathrm{N_0}$$

Thus, 90.25% of original amount will remain after 100 years.

5. CONCLUSION

Yang Transform is successfully used to find solution of linear differential equations satisfied by Population Growth and Radioactive Decay Problems.

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