CHOICE OF TOOTHPASTE BRAND FOR ORAL HEALTH BY USING FUZZY DECISION MODEL

C. G. Magadum[1], M. C. Naik[2]
Department of Mathematics[1], Department of Chemistry[2]
Smt. KasturbaiWalchand College, Sangli[1, 2]

Abstract - In market various toothpaste brands are available. Every consumer gives importance to healthy tooth and gums, prevention of tooth decay, herbal attribute, whiteness, long lasting freshness, good foam and price. Actual percentage of ingredients in various toothpaste brands are aggregated with respect to permissible theoretical percentage in toothpaste. Permissible theoretical percentage indicates the relative importance of the ingredients type in toothpaste.A λ-fuzzy measure is evaluated to obtain the index. Comparing indices the toothpaste brands are ranked.

Keywords - Fuzzy measure, Fuzzy integral, Aggregation operator, Ranking

1. Introduction: Inorganic and organic different multi-component mixtures are used for making toothpastes. For any company consumer's satisfaction is the target because he is the king of any products made by the company. Success rate of company is depended on consumer's behavior and satisfaction. Oral care product manufactures are well aware of the consumer dissatisfaction with their perceived tooth color and, in response, have developed a vast choice of contemporary toothpastes to address the problem [3]. In the formulation of toothpaste the most common functional ingredients are abrasives, active agents, flavoring agents, whitening agents, foaming agents, sweetener, preservatives, filters, cleaning agents, rheology modifiers etc. Now the choice of toothpaste varies from person to person depending upon the ingredients.

A new approachfor the analysis of toothpaste is determined by using fuzzy decision model in the present paper. Classical methods are expressed numerically and based mainly on a quantitative approach. Fuzzy logic theory is the branch of mathematics which concerns with a degree of uncertainty. Hence, it is useful in solving real-world problems.

Here, actual percentage of ingredients in various toothpaste brands is aggregated with respect to permissible theoretical percentage in toothpaste. The relative importance of the ingredients type in toothpaste is represented Permissible theoretical percentage. The fuzzy indices are calculated using λ-fuzzy measure which is evaluated by using mathematical programming. Comparing indices the toothpaste brands are ranked.

2. Literature review: Several thousand years ago, toothpaste formulation contains suspensions of crushed egg shells or ashes to complex formulation with often more than 20 ingredients. Around 3000-5000 BC, ancient Egyptians first developed a dental cream which contained powdered ashes from oxygen hooves, myrrh, eggshells and pumice to remove debris from teeth[7].Periodontal diseases(also called periodontitis) are those diseases that affect one or more of the periodontal tissues: alveolar bone, periodontal ligament, cementum and gingiva and occurs when bacteria in plaque infect the gums and bones that anchor the teeth [2]. So periodontitis is the primary cause of adult tooth loss[2].

Evaluation of the results showed variations between the classical and fuzzy logic methods [1]. Although performance evaluation using fuzzy logic is complicated and requires additional software, it provides some evaluation advantages [1]. Fuzzy logic evaluation is flexible and provides many evaluation options, while the classical method adheres to constant mathematical calculation [1].

This study proposes a new evaluation method for choice of toothpaste which is based on fuzzy logic systems.

3. Methodology: In decision modeling information fusion is most essential to combine data obtained from various sources. Aggregation operators play a vital role in fusion of information. In fuzzy decision theory, the fuzzy integrals like Choquet integral, Sugeno integral etc. can be worked as aggregation operators. In this theory we have to obtain aggregation of the preference values or satisfaction degrees.Common aggregation operators like arithmetic mean, weighted mean, median, mode etc. have some drawbacks because they only express the quantitative approach. But to express the qualitative approach like relation between criteria, decision making etc. we need fuzzy integrals. These integrals help in fusion of information and data mining effectively. The present study is based on aggregation operators such as Choquet integral, Sugeno integral and weighted arithmetic mean.

3.1 Sugeno's λ –fuzzy measure:

Let $\lambda \in (-1, \infty)$. A normalized set function g_λ defined on 2^Θ is called as λ –fuzzy measure on Θ if for every pair of disjoint subsets $\theta_1 \text{ and } \theta_2$ of Θ we have

$$g_\lambda(\theta_1 \cup \theta_2) = g_\lambda(\theta_1) + g_\lambda(\theta_2) + \lambda\, g_\lambda(\theta_1).\ g_\lambda(\theta_2)$$

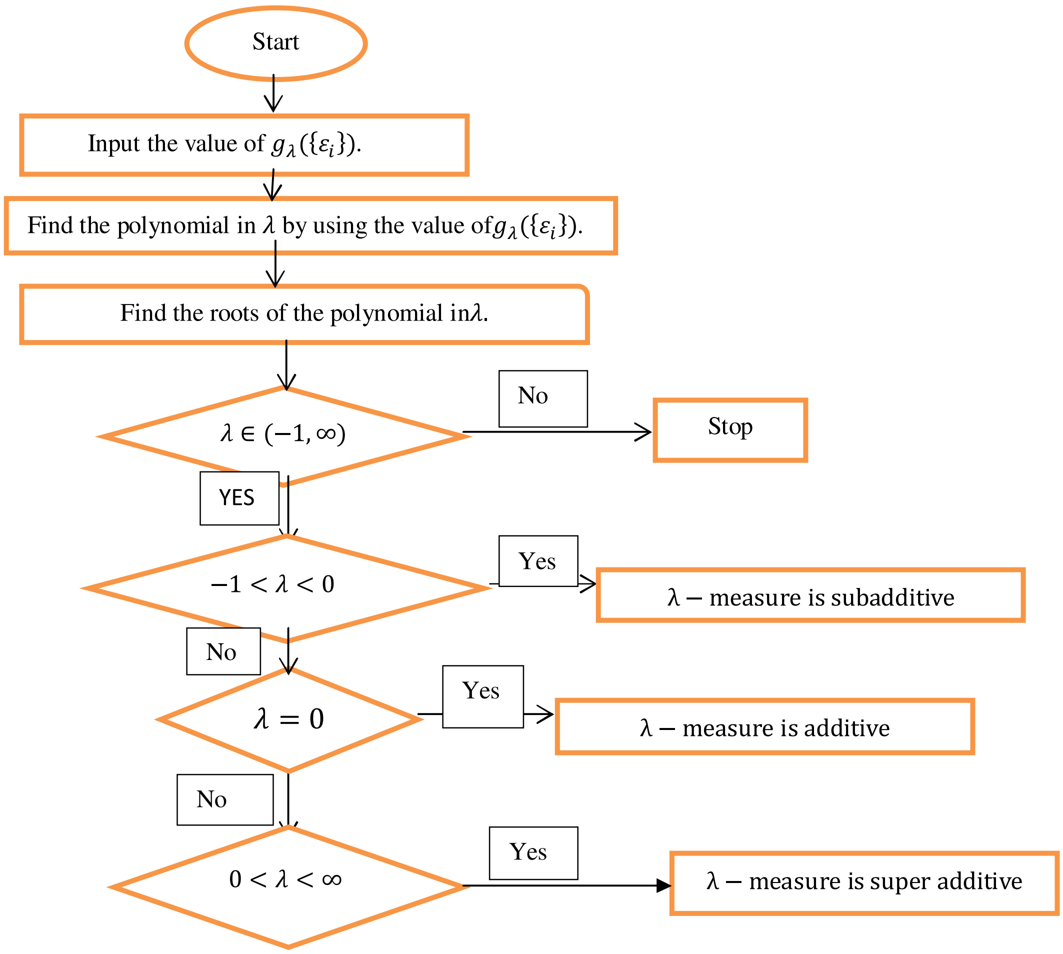
Obviously if $\lambda = 0$, then a λ-fuzzy measure is a normalized additive measure i.e. probability measure. A Dirac measure is a λ-fuzzy measure for all $\lambda > -1.$ This is the monotone measure [8].By following theorem the parameterλ is calculated [8].

3.1.1 Theorem: Let Θ be the finite set, $\Theta = \{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\}$ and 2^ϵ be the class of all subsets of Θ, the fuzzy measure $g_\lambda(\Theta) = g_\lambda(\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\})$ can be formulated as

$$g_\lambda(\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\}) = \frac{1}{\lambda}\left[\prod_{i=1}^{n}[1 + \lambda\, g_\lambda(\{\varepsilon_i\})] - 1\right] \text{ where } \lambda \in (-1, \infty)$$

As $g_\lambda(\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\}) = 1$ the formula becomes $\lambda + 1 = \prod_{i=1}^{n}[1 + \lambda\, g_\lambda(\{\varepsilon_i\})]$.[8]

3.2 Flow chart of calculation of λ: A Mathematical program is used to calculate λ –fuzzy measure.



3.3 Choquet Integral : Let f be a nonnegative measurable function on (Θ, B). The Choquet integral of f with respect to g_λ is denoted by $C_{g_\lambda}(f) = \sum_{i=1}^{n}(f(\varepsilon_i) - f(\varepsilon_{i-1}))\, g_\lambda(\theta_i)$ where $\theta_i = \{\varepsilon_i, \varepsilon_{i+1}, \dots, \varepsilon_n\}$, $f(\varepsilon_0) = 0$ and $(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)$ is a numbering of the elements of Θ satisfying the condition that $f(\varepsilon_1) \le f(\varepsilon_2) \dots \le f(\varepsilon_n)$.

3.4 Sugeno Integral: Let g_λ be a normalized fuzzy measure on Θ and f be a function on (Θ, B) with range $\{f(\varepsilon_1), f(\varepsilon_2) \dots, f(\varepsilon_n)\}$ where $0 \le f(\varepsilon_1) \le f(\varepsilon_2) \dots \le f(\varepsilon_n) \le 1$. The Sugeno integral $S_{g_\lambda}(f)$ with respect to measure g_λ is defined as $S_{g_\lambda}(f) = \vee_{i=1}^{n}[f(\varepsilon_i) \wedge g_\lambda(\theta_i)]$, where $\theta_i = \{\varepsilon_i, \varepsilon_{i+1}, \dots, \varepsilon_n\}$.

4. Case Study:

In market a large number of toothpaste are available. Consumer has to decide the best toothpaste for the oral care in his budget. Here five brands of toothpaste are considered. Six characteristics of toothpaste are considered namely abrasive, fluoride components (active ingredients), flavoring agents, whitening agents, cleaning agents and price. The grades of importance of each characteristic are given below.

$g_\lambda(x_1) = g_\lambda(\{abrasive\}) = 0.8$

$g_\lambda(x_2) = g_\lambda(\{Fluoride\ components\}) = 0.6$

$g_\lambda(x_3) = g_\lambda(\{Flavoring\ agents\}) = 0.5$

$g_\lambda(x_4) = g_\lambda(\{Whitenig\ agents\}) = 0.7$

$g_\lambda(x_5) = g_\lambda(\{Cleaning\ agents\}) = 0.7$

$g_\lambda(x_6) = g_\lambda(\{Price\}) = 0.5$

The details of ingredient type of five brands are shown in Table 1.The details of characteristics of toothpaste brands under consideration are mentioned Table 2.

Table 1: Ingredients type in five toothpaste brands

Brand	Abrasive	Fluoride components	Flavoring agents	Whitening agents	Cleaning agents
B1	Calcium Carbonate	Sodium fluoride	Sodium Saccharin	Hydrogen Peroxide	SLS
B2	Calcium carbonate, Hydrated silica	Sodium fluoride	Mint	Titanium dioxide	SLS
B3	Sodium Bicarbonate	Non fluoridated	Clove oil	Titanium dioxide	SLS
B4	Calcium pyrophosphate	Sodium mono fluorophosphates	Menthol	Titanium dioxide	SLS
B5	Sodium bicarbonate	Sodium mono fluorophosphates	Peppermint	Carbamide Peroxide	SLS

Table 2: Percentage of ingredient components in each brand

Brand	Abrasive (in %) X1	Fluoride components (%) X2	Flavoring agents(%) X3	Whitening Agents(%) X4	Cleaning Agents(%) X5	Price (in $) X6
B1	30	0.243	1.78	0.098	2.25	1.1607
B2	45	0.3678	0.798	0.1	1.75	0.4285
B3	25	0.0	1.98	0.1	1.5	0.2857
B4	20	0.184	1.85	0.0768	2.15	1.1363
B5	38	0.275	0.945	0.084	1.35	0.3759

By using mathematicalprogramming we have a polynomial in λ as

$$2.8x + 5.98x^2 + 4.988x^3 + 2.3257x^4 + 0.5747x^5 + 0.0588x^6 = 0$$

The roots of above equation are $0, -0.9981593, -1.2040376 + 1.6762409i,$

$1.2040376 \quad 1.6762409i$, $3.1837875 \mid 1.0313252i$, $3.1837875 \quad 1.0313252i$.
Here the complex roots are discarded and $\lambda = 0$ gives the additive measure. Therefore only $\lambda = -0.9981593$ is considered for calculation.

Table3: Fuzzy measures of characteristics and their permutations

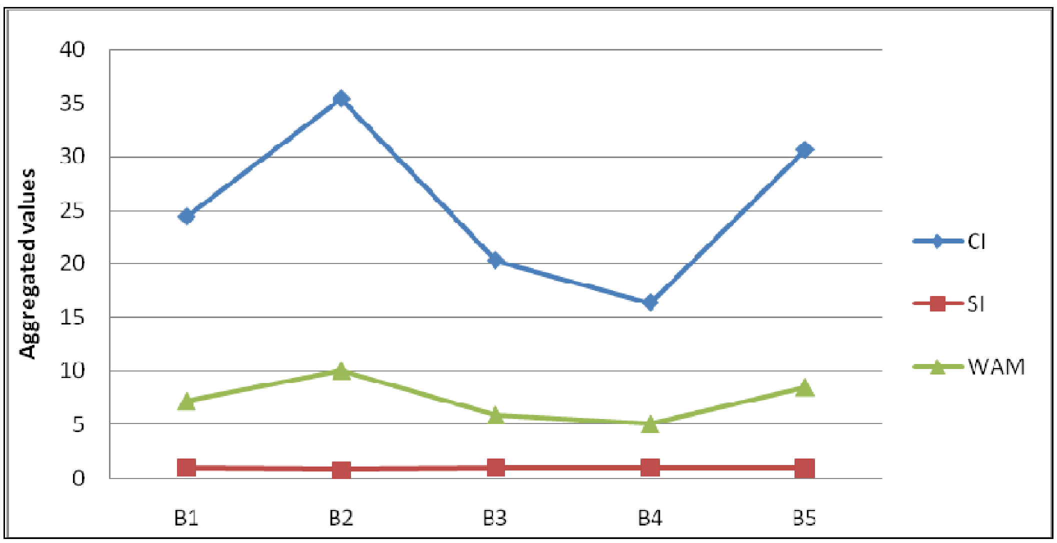
Between two criteria	λ-measure	Among three criteria	λ-measure
x_1,x_2	0.9208835	x_2,x_4,x_5	0.9653668
x_1,x_3	0.9007363	x_2,x_4,x_6	0.9411972
x_1,x_4	0.9410308	x_2,x_5,x_6	0.9411972
x_1,x_5	0.9410308	x_3,x_4,x_5	0.9562893
x_1,x_6	0.8005522	x_3,x_4,x_6	0.9261050
x_2,x_3	0.8807731	x_3,x_5,x_6	0.9261050
x_2,x_4	0.8807731	x_4,x_5,x_6	0.9562893
x_2,x_5	0.8005522	*Among four criteria*	*λ-measure*
x_2,x_6	0.8506442	x_1,x_2,x_3,x_4	0.9896254
x_3,x_4	0.8506442	x_1,x_2,x_3,x_5	0.9896254
x_3,x_5	0.7504602	x_1,x_2,x_3,x_6	0.9815294
x_3,x_6	0.7504602	x_1,x_2,x_4,x_5	0.9944949
x_4,x_5	0.9109019	x_1,x_2,x_4,x_6	0.9896254
x_4,x_6	0.8506442	x_1,x_2,x_5,x_6	0.9896254
x_5,x_6	0.8506442	x_1,x_3,x_4,x_5	0.9926661
Among three criteria	*λ-measure*	x_1,x_3,x_4,x_6	0.9865847
x_1,x_2,x_3	0.9612893	x_1,x_3,x_5,x_6	0.9865847
x_1,x_2,x_4	0.9774516	x_1,x_4,x_5,x_6	0.9926661
x_1,x_2,x_5	0.9774516	x_2,x_3,x_4,x_5	0.9835719
x_1,x_2,x_6	0.9612893	x_2,x_3,x_4,x_6	0.9714648
x_1,x_3,x_4	0.9713815	x_2,x_3,x_5,x_6	0.9714648
x_1,x_3,x_5	0.9713815	x_2,x_4,x_5,x_6	0.9835719
x_1,x_3,x_6	0.9511971	x_3,x_4,x_5,x_6	0.9790248
x_1,x_4,x_5	0.9835217	*Among five subjects*	*λ-measure*

x_1,x_4,x_6	0.9713815	x_1,x_2,x_3,x_4,x_5	0.9981627
x_1,x_5,x_6	0.9713815	x_1,x_2,x_3,x_4,x_6	0.9957235
x_2,x_3,x_4	0.9411972	x_1,x_2,x_3,x_5,x_6	0.9957235
x_2,x_3,x_5	0.9411972	x_1,x_2,x_4,x_5,x_6	0.9981627
x_2,x_3,x_6	0.9010129	x_1,x_3,x_4,x_5,x_6	0.9972466
		x_2,x_3,x_4,x_5,x_6	0.9926912

Table 4 : Scores for Choquet integral, Sugeno integral and Weighted arithmetic mean for each brand

Brand	Choquet integral	Sugeno integral	Weighted arithmetic mean
B1	24.39163	0.9865847	7.173618
B2	35.385466	0.8	10.03393
B3	20.310835	0.9713815	5.856013
B4	16.37865	0.9865847	5.042713
B5	30.62766	0.945	8.481382

Figure1: Aggregated values of each brand by using Choquet integral(CI), Sugeno integral(SI) and weighted arithmetic mean(WAM)



The toothpaste brand ranking according to Choquet integral $B2 \succ B5 \succ B1 \succ B3 \succ B4$.
The toothpaste brand ranking according to Sugeno integral $B4 \succcurlyeq B1 \succ B3 \succ B5 \succ B2$.

From table 4, the Choquet integral gives fine variations in each brand. In other hand the Sugeno integral do not such variation at the brand B4 and B1. Also, the WAM do not focus on each criteria effectively.

5. Conclusion:

In real decision making problems, the usual aggregate operators are not enough to understand the complex situations. In such circumstances fuzzy decision model play an important role. In the present study the permissible actual values of ingredients in toothpaste are modeled using novel Choquet integral and Sugeno integral. Choquet integrated values shows large deviations than that of Sugeno integral and WAM values. By using fuzzy decision model we can conclude that Choquet integral gives better values than SI and WAM for the choice of toothpaste brand for oral health.

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