VARIANCE – COVARIANCE (DELTA NORMAL) APPROACH OF VAR MODELS:AN EXAMPLE FROMBOMBAY STOCK EXCHANGE

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Abstract

Numerous investors are inclined to understand, the quantum of wealth or capital they can lose in a specific time period, which could be one day or 5 days or 10 days.In this research paper, out of numerous approaches, variance – covariance approach of VaR is discussed.This method helps in prediction of maximum loss that can occur for a specific time period and given probability. Here in order to calculate VaR, portfolios are created, which is followed by identification of returns distribution. Finally VaR of portfolios is calculated. Daily loss is calculated using data for the period of 01st January 2018 to 31st December 2018as historical data consisting of 246 days. Companies were selected from Bombay Stock Exchange (BSE). VaR has been computed for both 95% and 99% confidence intervals for holding period of 1 day and 10 days.

Keywords : Risk & Return, VaR, Maximum Loss, Variance – Covariance approach, Correlation.

1. Introduction

The key yardsticks or benchmark for any investment happens to be Risk and Return. The primary assumption w.r.t investors are that they want to maximize their utility and reduce risk. In other words they are risk averse. From that perspective, risk management has emerged as a vital facet for evaluation and choice of investments. Numerous organizations and regulatory authorities too attribute tremendous significance to analysis and measurement of risk in the aftermath of 2008 financial crisis.

A number of methods exist to compute market risk, which primarily consists of exchange rates, interest rates etc. Value at Risk models (VaR) have been implemented since 1994. The primary disadvantage of VaR models is that they estimate or predict only those risks that can be quantified. Qualitative risks such as regulatory risk, operational risk or political risks cannot be measured using this model.VaR model estimates the highest loss for a portfolio or a stock w.r.t a given holding period and a level of confidence. This gives investors an opportunity to know, how much they stand to lose in a particular time frame.

Different results are thrown up by different models.VaR models have both advantages as well as disadvantages. The classification of VaR models can be parametric or non – parametric. Variance – Covariance model (also known as delta normal) is the most widely used VaR model in finance.

The advantage of using this model is that it is very simple and straight forward to use. This model uses covariance and correlation matrices to measure standard deviation and variance of a risky asset portfolio. This model goes with a primary assumption that returns should follow a normal distribution.

Research Objective: -

The primary objective of this paper is to elaborate on Variance – Covariance VaR method and to measure the maximum loss that an investor can incur on a portfolio comprising of a variety of stocks. The research paper is divided into following parts: -

A brief introduction of VaR models in given in Part 2. In Part 3, the approach of VaR – CoVaR is applied to measure loss value. A number of statistical techniques are applied in this section. Finally the paper is concluded in section 4.

The aim of this paper is briefly to elaborate on Variance-Covariance method and tocompute maximum risk for portfolios consisting of different stocks. The rest of the paper is organised as follows. Inpart 2, VaR models are briefly introduced. In part 3, variance-covariance method is applied to calculate loss value. Many statistical techniques are used in this section. Finally, part 4 concludes thepaper.

2. Variance-Covariance Approach of VaR Methods

Every investor wants to earn maximum returns at minimum risk. Therefore, prediction and estimation of risk is a very important parameter in all decisions related to investments. The investor will be willing to invest in any portfolio or asset only if the expected return is more than its perceived cost. Normally there is a compromising situation or a trade – off situation experienced by every investor, wherein a good but relatively small investment in terms of money may result in an opportunity cost, however a substantially large but bad investment may lead to enormous losses. Therefore risk management becomes imperative and is inevitable to attain the stated objective of maximum return at minimum risk. Value at Risk (VaR) is a popular method of risk measurement to compute the worst case scenario loss at a specific level of confidence over a specific time frame(Johansson, 2013).

VaR was applied for the first time by J.P Morgan in 1994 creating CreditMetricsmethodology, RiskMetrics and RAROC models. This model later on has been applied by numerous other organizations. (Anjunas, 2009)

Even though regulatory groups have been pushing it vigorously for setting regulatory minimum capital standards, numerous financial companies too have devised their own methods based on VaR as a medium to manage and monitor market risk(Darbha, 2001). In 1995, Basel Committee of Banking Supervision, USA Federal Reserve System and USA Stock Committee and in 1996, European Union Capital Requirements Directive put forward a proposal to implement Value – at – Risk method as one of the tool for market risk management(Anjunas, 2009).

VaR models have three primary assumptions: -

1) Static requirement meaning that daily fluctuations of returns are completely independent and unrelated from previous day's or future day's return. It is associated with random walk theory in finance.
2) The second assumption is that of non – negativity. It implies that the financial assets do not have negative values.
3) VaR model assumes that the historical data w.r.t stock/portfolio returns is normally distributed (Allen, 2004)

VaR approaches can be primarily categorized into two categories.

1) Parametric (Variance – Covariance, also known as delta normal)
2) Non – Parametric (This consists of two simulation methods, which are called as historical simulation and Monte Carlo simulation). Both methods have their own advantages and disadvantages(Bozkaya, 2013).

In this research paper, preference is accorded to Variance – Covariance approach to compute the portfolio loss. Ease of application and nimbleness are the two distinct pluses of this method. Also it is not imperative to assume that distribution of returns is static through time as volatility has been incorporated into estimation of parameters. (Bohdalova, 2007).

The main disadvantage of this method amongst other disadvantages is that it is tremendously dependent on the assumption that returns are normally distributed, however evidence suggests that market returns generally have "fat tails" and do not look like a true normal distribution.

3. Research Methodology

3.1. Data and Formulas

In this research paper, historical data has been employed by Variance – Covariance approach to compute maximum (worst case) loss for a certain given confidence interval over a certain holding time period. Therefore confidence interval and holding time period are the primary yardsticks of measurement.

In this study, two hypothetical portfolios are created at first. They have same three companies'stocksbut different weights. These three companies trade in Bombay Stock Exchange (BSE). They are operatingin industries as given below.

Table 1: - Name and Industry of Companies

Name of the Company	Industry
Mahindra & Mahindra Ltd	Automobile
Wipro Ltd	IT
ICICI Bank	Banking

`In this research paper, beta of companies and market return are estimated using daily returns (Adjusted price for INR). They have been borrowed from BSE Database. One year period (246 working days) data is employed. The period is from) 01st January 2018 to 31st December 2018. Also BSE SENSEX has been used as market index.

To calculate for stocks daily return; the formula is applied as follows:

$$R_i = \frac{R_{it} - R_{it-1}}{R_{it-1}}$$

Where "Ri" is a daily return of share i, "Rit" is a closing price of share i in t date and "Rit-1" is a closing priceof share i in t - 1 date

To Calculate the index (BSE SENSEX) daily return, the following formula is applied.

$$R_{BSE} = \frac{BSE_t - BSE_{t-1}}{BSE_{t-1}}$$

Where $R_{BSE}$is average return of market. $BSE_t$is a market return in t date. $BSE_{t-1}$is a market return in t-1 date.

In order to calculate variance of stocks daily return and index return, I used the following historical volatility formula.

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Ri - Raverage)^2$$

Where "σ^2" is a variance of daily share return. Ri is a daily return of share i. Raverage is the average daily return. N is a sample size (246) days.

To measure, how stocks vary together, standard formula for covariance can be used.

$$\text{Cov (X,Y)} = \frac{1}{n-1}\sum_{i=1}^{n}[(X_i - \bar{X}).(Y_i - \bar{Y})]$$

where the sum of the difference of each value X and Y from the average is then further divided by the total number of values minus one. The covariance calculation enables us to calculate the correlation coefficient, shown as:

$$\text{Correlation Coefficient} = \frac{Cov(X,Y)}{\sigma_X.\sigma_Y}$$

Where σ is the standard deviation of each asset/stock. However, if there are more than two financial assets in the portfolio, then correlation and covariance matrices are needed to solve equations. To calculate standard deviation of portfolio (position) the following formula is used:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}(w_i^2.\sigma_i^2) + 2(\sum_{i=1}^{n}\sum_{j=1}^{n}(w_i.\sigma_i.w_j.\sigma_J.\rho ij)}$$

Where "σ_p" is a standard deviation of portfolio, "σ_i" is a standard deviation of stocks. Wi is a weight of stocks in a portfolio and Pij is a correlation coefficient between stocks i and j.

3.2. Analysis and Interpretation

In this study, excel functions and data solver are used for all calculation. The calculation of variance- covariancemodel involves the following steps:

Step 1 – Determining period to hold and Confidence Level (table 2)

Step 2 – Determining Portfolio (table 3 and table 4)

Step 3 – Statistical Details of Returns (table 5)

Step 4 – Determining Correlations and Covariance between Assets (table 6 and table 7)

Step 5 - Calculating the Volatility of the Portfolio (table 8)

Step 6 - Calculating the VaR Estimate (table 9)

Table 2: - Main Parameters of Calculations

Parameter	Value
Confidence Level	**95% and 99%**
Time Horizon	**1 Day and 10 Days**
Size of Historical data	**246 Days**
Testing Period	**01 – 01 – 2018 to 31 – 12 – 2018**

Closing price (as on 31 – 12 – 2018) of stocks along with weights are given below in Portfolio 1 and Portfolio 2. They have same stocks butdifferent weights.

Table 3: - Portfolio 1

Stocks	Closing Price (31 – 12 – 2018) in Rupees	Weights
M&M Ltd	**803.700**	**33.3%**
WIPRO Ltd	**330.25**	**33.3%**
ICICI Bank	**360**	**33.3%**

Table 4: - Portfolio 2

Stocks	Closing Price (31 – 12 – 2018) in Rupees	Weights
M&M Ltd	**803.700**	**50%**
WIPRO Ltd	**330.25**	**30%**
ICICI Bank	**360**	**20%**

Table 5: - Statistical Details

Stocks	BSE SENSEX Returns	M&M Ltd Returns	WIPRO Ltd Returns	ICICI Bank Returns
Minimum	-0.02366941	-0.068921185	-0.043191939	-0.061096253
Average	0.00026358	0.000339849	0.000170777	0.00066673
Maximum	0.021312587	0.051512356	0.04766844	0.102770294
Variance	0.00006	0.00028	0.00020	0.00039
Standard Deviation	0.007939661	0.016699341	0.014092362	0.019741286

Variance – Covariance employs matrix calculations in order to compute Value – at – Risk for portfolios comprising of numerous stocks/assets. As seen from Variance-covariance approach uses matrices giving chance to measure VaR value for a portfolioconsisting of hundreds of assets. As we can see from the formula for portfolio measurement, standard deviation of portfolio calculation warrants computation of correlations of each asset and also covariance between them.Also application of variance covariance matrices is a pragmatic method of computing standard deviation of portfolio. In this research, the objective is to show, how the parametricmethodology utilizes variance and correlation matrices to calculate the variance, and hence standard deviation, of aportfolio.

Table 6: - Correlation Matrix

Correlation Matrix	BSE SENSEX	M&M Ltd	WIPRO Ltd	ICICI Bank
BSE SENSEX	1			
M&M Ltd	0.59453	1		
WIPRO Ltd	0.602216	0.441329	1	
ICICI Bank	0.696572	0.392052	0.404383	1

The strength of association between two variables is computed through correlation. This tells us the quantum in terms of percentage and direction in which the two variables move together. Volatility of a portfolio or the risk of a portfolio is lower than its individual assets volatility or risks.

Therefore it is imperative to know the relation between variance of a portfolio and variances of assets. Correlation always has a value between -1 and +1(Bozkaya, 2013).As we can observe from Table 6, correlation between stock returns of Bharti Airtel Ltd and Maruti Suzuki Ltd is higher than correlation between HDFC Bank Ltd and Maruti Suzuki Ltd and also higher than correlation between HDFC Bank Ltd and Bharti Airtel Ltd. But all correlations are positive.

The coefficient of correlation may be computed by employing the covariance matrix that measures how mean returns of two companies vary or move together. Covariance matrix assists decision makers in deciding on the assets that move in the same direction or in the opposite direction(Bozkaya, 2013).

Table 7: - Covariance Matrix

Covariance Matrix	BSE SENSEX	M&M Ltd	WIPRO Ltd	ICICI BANK LTD
BSE SENSEX	**6.27809E-05**	**7.53537E-05**	**2.49809E-05**	**8.62372E-05**
M&M Ltd	**7.53537E-05**	**0.00027773**	**1.89357E-05**	**0.000102151**
WIPRO Ltd	**2.49809E-05**	**1.89357E-05**	**0.000197784**	**3.9046E-06**
ICICI BANK LTD	**8.62372E-05**	**0.000102151**	**3.9046E-06**	**0.000388128**

Covariance enables us to compute volatility of portfolios. Covariance values between stocks are multiplied by each shares weights and then added to find volatility of portfolio.

Table 8: - Standard Deviation and Variance of Portfolio 1 and Portfolio 2

	Portfolio 1	Portfolio 2
Standard Deviation	0.01101243	0.0113727
Variance	0.000121	0.000129

VaR is calculated as using the following formula:

P*α*σ*sqrt(t)

Where "P" is the value of portfolio (or position), **"α"**is confidence level, **"σ"**is a volatility of portfolio and t is a holding period. For 95% level of confidence, **"α"** is 1.65 and for 99% level of confidence**"α"** is 2.33.VaR may be calculated for different time periods. In this research both 1 day

and 10 day periods have been considered. If someone needs to determine VaR values for more than 1 day, such as 10 days, they need to multiply daily volatility returns by sqrt(10). Here if we consider principal value **P** as 1000000 (10 Lakhs), then the VaR values are as follows:

Table 9: - VaR Values of Portfolio 1 and Portfolio 2

	Portfolio 1		**Portfolio 2**	
	95%	**99%**	**95%**	**99%**
VaR (One Day)	18170.51	25658.96	18764.9	26498.31
VaR (Ten Days)	57460.2	81140.76	59339.82	83795.02

Market risk of portfolios can be computed through VaR models. As we can see from the above table, at 95% level of confidence and time period of 1 day, maximum loss that portfolio 1 can incur is Rs 18170.51and maximum loss that portfolio 2 can incur is Rs18764.9. It indicates that there is only 5% probability that loss for the next day will be greater than Rs 18170.51for portfolio 1 and Rs 18764.9for portfolio 2. At Confidence level of 95% and time period of 10 days, maximum loss will not be more than Rs 57460.2 for portfolio 1 and not more than Rs 59339.82for portfolio 2.

At 99% level of confidence and time period of 1 day, maximum loss that portfolio 1 can incur is Rs 25658.96and maximum loss that portfolio 2 can incur is Rs 26498.31. It indicates that there is only 5% probability that loss for the next day will be greater than Rs 25658.96for portfolio 1 and Rs 26498.31 for portfolio 2. At Confidence level of 99% and time period of 10 days, maximum loss will not be more than Rs 81140.76 for portfolio 1 and not more than Rs 83795.02for portfolio 2.

Conclusion

It is possible for investors to estimate probable loss value of their portfolios for different holding periods andconfidence level. Variance–covariance approach helps us to measure portfolio risk if returns are distributednormally. In this study, two hypothetical portfolios to calculate potential loss with both 95% and 99% confidencelevel as well one day and ten days holding periods are created. As a main conclusion, there is no huge differencebetween Portfolio 1 and Portfolio 2 results. It is thought that the portfolio was not diversified well. There wereonly three stocks in the portfolio but importantly their correlations were not low enough to decrease risk adequately.Stocks have equal weights in Portfolio1 as 33.3 %. Stocks have different weights in Portfolio 2. Thus, while risk evaluationof one stock is related with especially volatility characteristics, risk evaluation of portfolio is related withcorrelation between risky assets inside the portfolio.

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